The complex problem of the complexity of Minimum dominating set algorithms

Dr. M. Evangeline Prathibha
Assistant Professor
Department of Mathematics
Lady Doak College,
Madurai. Tamil Nadu

Basics

- Domination, a fundamental concept in Graph Theory, seeks to control a network by strategically positioning a minimal set of “domineering” elements.
- Dominating sets are important for large-scale scheduling as well as efficient resource allocation.
- The algorithm to determine the dominating set of a graph is found to be NP-hard.
- The complexity of an algorithm refers to the amount of resources(such as time or memory) required to solve a problem or perform a task.

- The classical bound in complexity so far in a minimum dominating set is $O(1.5137)^n$
- This was discovered by Van Rooji, Nederlof J and Van Djik [5] in 2009.
- We have many advances since then, with many algorithms being tailor -made to find minimum dominating sets for Ad-hoc networks[7], for specific kinds of massively parallel architectures[8] etc.
- One approach is to make the NP complete problem fixed parameter tractable.

- A problem is called fixed - parameter tractable if it can be solved in $f(k)n^{O(1)}$for an arbitrary function f which depends only on k. According to the theorem of parametrized complexity, it is very unlikely that minimum dominating sets are fixed-parameter tractable.
- However planar dominating sets are found to be fixed- parameter tractable[2] and the complexity which was found to be $O(11^k n)$ (where n is the number of vertices) was improved to $O(c\sqrt{k})^n$ for some constant c.

A minimum connecting dominating set (MCDS) is essential for a wireless sensor network to conserve its energy and send messages in an optimized way.

Using pseudo dominating sets, an MCDS[7] was constructed by first constructing a pseudo dominating set (PDS), then by an improved Steiner Tree construction technique, the nodes of PDS are connected after which we remove redundant vertices and then we arrive at an MCDS.

The message and time complexity of this algorithm is O (n)

and O (D) respectively, where n is the network size, D is

the network diameter and is the maximum degree of all the nodes.

The performance ratio of this algorithm is (4 . 8 + ln 5) loptl + 1 . 2, where l opt l is the size of any optimal CDS.

New Break through

- There is now a biomolecular and quantum algorithm discovered in 2023 for finding the dominating set in arbitrary networks and that now claims to have the best complexity known to man.
- The complexity of this algorithm is $O(2^{n/2})$ quantum gates, where n is the number of vertices of G.
- This has been tested on quantum simulators like the IBM quasm simulator and the Brooklyn(a 65 qubit system).

Question(s)

- Given that, all the different approaches to finding minimum dominating set, have been ”compromising” on one aspect or the other, and what works for one kind of system may not work for another, is the “tailor-made” domination algorithm the way to go forward, or should another approach be thought of?
- Since we now have quantum computing also entering into the realm of MDS algorithms, the quantum solutions that are promised are vastly improved when compared to traditional solutions, are we to abandon the usual methods?
- Mathematicians and Computer scientists have been constantly trying to reduce the complexity of MDS algorithms. Is there a lower bound that can be established which says, that the complexity cannot be improved further?

Bibliography

1. Michael R. Garey, David S. Johnson, *Computers and Intractability –A guide to the theory of NP-completeness*, W. H. Freeman and Company, New York.
2. J. Alber, H. L. Bodlaender, H. Fernau, R. Niedermeier, *Fixed parameter algorithms for planar dominating set and related problems*, in: Scandinavian Workshop on Algorithm Theory, Springer, 2000, pp. 97–110.
3. M. H. Nguyen, M. H. H`a, D. N. Nguyen, T. T. Tran, *Solving the k-dominating set problem on very large-scale networks*, Computational Social Networks 7 (2020) 1–15.
4. A. Mun˜oz, F. Rubio, *Evaluating genetic algorithms through the approximability hierarch*y, Journal of Computational Science 53 (2021) 101388.
5. J. M. Van Rooij, J. Nederlof, T. C. van Dijk, *Inclusion/exclusion meets measure and conquer: Exact algorithms for counting dominating sets*, in: European Symposium on Algorithms, Springer, 2009, pp. 554–565.
6. R. Wong, W.-L. Chang, W.-Y. Chung, A. V. Vasilakos, *Biomolecular and quantum algorithms for the dominating set problem in arbitrary networks,* Scientific Reports 13 (1) (2023) 4205.
7. Jasawi Prasad Mohanty, Chittaranjan Mandal, Chris Reade, Ariyam Das, *Construction of minimum connected dominating set in wireless sensor networks using pseudo dominating set*, Ad Hoc Networks 42 (2016) 61–73.
8. Evangeline Prathibha, S. Pious Missier, A. Anto Kinsley, *Algorithms to determine the distance-t-dominating sets of ES(n, k),* Procedia Computer Science (47) (2015) 342-350.