CALDAM 2024

Indo-Slovenia Pre-Conference School on Algorithms and Combinatorics

February 12 - 13, 2024

Young Researchers' Forum

Booklet of Abstracts

Organisers

Indian Institute of Technology Bhilai

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CALDAM 2024 Pre-Conference School on Algorithms and Combinatorics

February 12-13, 2024

Organisers :

- Association for Computer Science and Discrete Mathematics (ACSDM)
- Indian Institute of Technology Bhilai, Chhattisgarh, India.
- IIT Bhilai Innovation And Technology Foundation (IBITF)

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- Manoj Changat, University of Kerala, India
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- Rishi Ranjan Singh, Indian Institute of Technology Bhilai, India (Co-Convenor)

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Preface

Discrete Mathematics is a branch of mathematics involving discrete and distinct mathematical objects. It is viewed as one of the fundamental fields to understand the mathematical grounds of Computer Science, quintessentially Algorithms, Cryptography, Graph Theory, Computational Geometry, and related disciplines. These fields are evergreen and prove to be highly promising due to immense research conducted around the globe. The Indo-Slovenia Pre-conference School is an initiative to bring together eminent experts in the fields of Discrete Mathematics to provide a forum for researchers to discuss the current advancements in Algorithms and Discrete Applied Mathematics. The knowledge dissemination and sharing will encourage inquisitive students to become prospective researchers.

Young Reseachers' Forum (YRF) is a pioneer effort in the series of CALDAM conference. YRF is part of CALDAM Pre-Conference School and is the fourth in the series of CALDAM conferences. The forum is a unique opportunity as an open discussion session for ambitious young researchers to present a problem they are attempting to solve in the theme of the conference and to receive feedback from participating peers and eminent researchers. YRF provides a friendly environment for young researchers to foster research. An interactive session of this kind enables students and researchers to engage in the exchange of knowledge, ideas, and research methods that benefit all participants of the Pre-Conference School. The deliberations with experts will be an inspiration and motivation to the students to pursue a research career in field of Algorithms and Combinatorics. The conference, along with the Pre-Conference School and Young Researchers' Forum it is associated with, provide a comprehensive platform that enables attendees to set off on an integrated journey of exploration and progress in the fascinating fields of algorithms and Combinatorics.

CALDAM Indo-Slovenia Pre-Conference School on Algorithms and Combinatorics

February 12-13, 2024

February 12, 2024		February 13, 2024	
08:30 - 09:15	Registration	-	-
09:15 - 09:30	Inaugural Session	-	-
09:30 - 10:30	**Technical Session – 1** Iztok Peterin Topic: On Properties of Modular and Direct-Co-Direct Products	09:30 - 10:30	**Technical Session – 5** Vesna Iršič Topic: Cops and Robber on Surfaces of Constant Curvature
10:30-11:00	Tea break		
11:00 - 12:00	**Technical Session – 2** Swagato Sanyal Topic: A Tutorial on Communication Complexity and its applications	11:00 - 12:00	**Technical Session – 6** Ragesh Jaiswal Topic: Some Results on Outlier and Constrained Clustering
12:00 - 13:00	Bodhayan Roy Topic: Conflict-Free Coloring of Polygons	12:00 - 13:00	Akanksha Agrawal Topic: Hybrid Parameterizations for Graph Problems
13:00 - 14:30	Lunch Break		
14:30 - 15:30	**Technical Session – 3** Riste Skrekoviski Topic: Selected Topics on Wiener Index	14:30 - 15:30	**Technical Session – 7** Tanja Dravec Topic: Grundy Domination Invariants
15:30 - 16:00	Tea Break		
16:00 - 18:30	**Technical Session – 4** Young Researchers' Forum (YRF 2024)	16:00 - 17:00	**Technical Session – 8** I. Vinod Reddy Topic: On Conflict-Free Coloring of Graphs
		17:00 - 17:15	Valedictory
19:30 - 21:00	Dinner		

CONTENTS

Two interesting transit functions from I_G

Lekshmi Kamal K. Sheela

Department of Futures Studies, University of Kerala, Trivandrum 695 581, India.
lekshmisanthoshgr@gmail.com

Abstract

The interval function I_G of a connected graph G is the function I_G from $V \times V$ to 2^V such that $I_G(u,v) = \{w \in V\colon w$ lies on some shortest u,v - path in $G\}$.

I_G satisfies the following classical axioms (b1), (b2), (b3) and (b4) on every graph G but (m) may not, see [2].

$(b1)$ If $x \in R(u,v)$ and $x \neq v$, then $v \notin R(u,x)$, $\forall u,v,x \in V$.
$(b2)$ If $x \in R(u,v)$, then $R(u,x) \subseteq R(u,v)$, $\forall u,v,x \in V$.
$(b3)$ If $x \in R(u,v)$ and $y \in R(u,x)$, then $x \in R(y,v)$, $\forall u,v,x,y \in V$.
$(b4)$ If $x \in R(u,v)$, then $R(u,x) \cap R(x,v) = \{x\}$, $\forall u,v,x \in V$.
(m) If $x,y \in R(u,v)$, then $R(x,y) \subseteq R(u,v)$, $\forall u,v,x,y \in V$.

Two interesting transit functions derived from the interval function are the cycle transit function C and the stress function S, defined as follows:

Cycle transit function: For $u,v \in V$, $\mathcal{C}(u,v) = \{u,v\} \cup \{w : w \in I(x,y)$ where $x,y \in I(u,v)$ and $I(x,y)$ induces a cycle $\}$. If $I(u,v)$ doesn't contain any cycle, then $\mathcal{C}(u,v) = \{u,v\}$.

Stress function: Let $u,v \in V$, $S(u,v) = \{x : x$ lies on every $(u,v)-$ shortest path$\}$.
Let $x \in I(u,v)$ and $x \notin S(u,v) \implies \exists$ more than one shortest u,v-path and x lies in only one $\implies x \in \mathcal{C}(u,v)$. So we have

Observation: For $u,v \in V(G)$, $I(u,v) = \mathcal{C}(u,v) \cup S(u,v)$.

Both the functions satisfies the axioms $(b1)$, $(b2)$, $(b4)$. From the definitions, we observe that both functions have contrasting behavior w.r.t the presence of distinct u,v-shortest paths. We note that $\mathcal{C}(u,v) = I(u,v)$ for all $u,v \in V(G)$ if and only if G is a thick graph (graph in which for any two vertices $x,y \in V$ with $d(x,y) = 2$ there exists at least two shortest paths between x and y) and $S(u,v) = I(u,v)$ for all $u,v \in V(G)$ if and only if G is a geodetic graph (a graph in which there is only one shortest path between any pair of vertices).

Problem Identify the properties satisfied by $\mathcal{C}$ but not S and vice-versa. Is it possible to characterize the function S using a set of first order axioms?

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A survey on Byzantine Gathering of Mobile Agents

Ashish Saxena

Indian Institute of Technology Ropar, India
ashish.21maz0004@iitrpr.ac.in

Abstract. The gathering of mobile agents in the presence of Byzantine faults is first studied by Dieudonné et al. [1]. Authors provide a polynomial time algorithm handling any number of weak Byzantine agents in the presence of at least one good agent considering start-up delays, i.e., the good agents may not wake up at the same time. Hirose et al. [2] come up with an algorithm considering start-up delays that use a strong team of at least $4f^2 + 8f + 4$ many good agents but runs much faster than that of Dieudonné et al.. Later Hirose et al. [3] provide another polynomial time algorithm for gathering in the presence of at least $8f + 8$ good agents. However, this algorithm does not work in the presence of start-up delays, and simultaneous termination of good agents is not possible. Recently, Saxena et al. [4] provide an algorithm considering start-up delays of the good agents reducing the number of good agents w.r.t. Hirose et al. from $4f^2 + 8f + 4$ to $f^2 + 4f + 9$. Also, their algorithm guarantees simultaneous termination of the good agents.

Dieudonné et al. [1] give algorithms that can handle strong Byzantine agents. However, their algorithms were not in polynomial time. Additionally, there was a gap between the upper and lower bounds for the number of agents in the presence of strong Byzantine agents. Bouchard et al. [5] later match the bounds for the number of agents but at the cost of an exponential time complexity. In more recent work, Bouchard et al. [6] introduced a polynomial time algorithm that can handle strong Byzantine agents, provided there are $5f^2 + 6f + 2$ good agents. Some authors [7], [8] have presented a polynomial-time algorithm that outperforms [6]. However, these algorithms make certain assumptions.

Keywords: Mobile agents, Anonymous graphs, Gathering, Byzantine Faults, Deterministic algorithm

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On Higher Multiplicity Hyperplane and Polynomial Covers for Symmetry Preserving Subsets of the Hypercube

Chandrima Kayal

Indian Statistical Institute, Kolkata

Abstract. Suppose we want to cover all the vertices of the n dimensional Boolean cube using minimum number of hyperplanes. Observe that this can be easily done using only two hyperplanes: any two hyperplanes containing two opposite $n-1$ dimensional faces are sufficient. Moreover, no single hyperplane can cover all the vertices. Now what if we want to cover only a subset of the Boolean cube? For example, suppose we want to cover all the vertices except one, viz. the origin. One can observe that n hyperplanes are sufficient. But can we do better? The celebrated covering result by Alon and Füredi [1] shows that at least n hyperplanes will be required to cover all the vertices of the n cube leaving out exactly one vertex. We shall discuss different versions of this covering problem, and we shall prove a generalization of Alon and Füredi's covering result for any symmetric subset of the Boolean cube. Also, we shall show a strict separation between the size of a polynomial cover and a hyperplane cover.
This work was jointly done with Arijit Ghosh, Chandrima Kayal and S. Venkitesh.

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Color-Coded Dispersion in Mobile Robot Networks

Himani

DA-IICT
Gandhinagar, Gujarat, India.

Abstract. The utilization of mobile robots to solve global problems in a distributed manner is a unique and intriguing approach to problem-solving. This approach can be applied to model numerous real-world problems, including toxic hazard cleanup, large-scale maze exploration, and collective gathering at a single location. The problem of dispersion was first introduced in [1] by Augustine and Moses Jr., where they had given a lower bound of $\Omega(\log n)$ on the memory and of $\Omega(D)$ on the time complexity for any deterministic algorithm on arbitrary graphs. The dispersion for arbitrary graph is widely studied in [1–3] for local and global communication model in [4].

We are currently focused on addressing the challenge of colored dispersion, where both nodes and robots exhibit distinct colors. *The color-coded dispersion problem on graphs is to disperse k colored robots among n colored nodes, ensuring each robot settles at a node sharing the same color*. In practical terms, this dispersion algorithm holds significant application, especially in contexts resembling multiple charge stations owned by different companies. For example, envision a scenario where electric cars or mobile robots navigate to their respective company's charge station. This application proves crucial in domains like logistics or transportation, where each colored node represents a distinct task or location. By coordinating spatial arrangement based on color-coded attributes, the algorithm enhances operational efficiency, proving valuable in dynamic environments where distinct colors signify specific tasks or locations.

Keywords: Dispersion · Time and message complexity · Mobile robots · Anonymous graphs

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ℓ-locating-dominating codes in infinite grids

Soura Sena Das

Indian Statistical Institute, Kolkata, India

Abstract. Precisely locating intruders in a facility or locating faulty processors in a multiprocessor environment has been the key motivation for locating-dominating codes, introduced by Slater in the late 80s. Locating multiple intruders or faulty processors in a system led to the generalization of locating-dominating codes into ℓ-locating dominating codes, introduced by Honkala. We study these codes for infinite grids and pose some open problems for the same.

Keywords: locating-dominating codes, ℓ-locating-dominating codes in grids.

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Laplacian State Transfer on Graphs

Swornalata Ojha

National Institute of Technology, Rourkela, Odisha-769008
ojhaswornalata9@gmail.com

Abstract. Let G be a finite, simple and undirected graph. A continuous-time quantum walk on G relative to the Laplacian matrix L is defined by the unitary matrix $U_L(t) := e^{itL}, t \in R$. A graph G is said to have Laplacian perfect state transfer (LPST) between two distinct vertices u and v if there is a time $\tau \in R$ such that $U_L(\tau)e_u = \gamma e_v$, for $\gamma \in C$ and $|\gamma| = 1$. LPST is a very rare phenomena, so we consider a relaxation to it which is known as Laplacian pretty good state transfer (LPGST). Then G is said to have LPGST between two distinct vertices u and v if there exists a sequence $\tau_n \in R$ and $\gamma \in C$ with $|\gamma| = 1$, such that $\lim_{n\to\infty} U_L(\tau_n)e_u = \gamma e_v$. The small graphs which exhibit LPST are K_2, C_4, $K_4\backslash e$. The complete graph K_{4n} with missing link has LPST between the two non-adjacent vertices. Also, it has been seen that every tree with atleast three vertices has no LPST. The LPGST occurs between the extremal vertices of the path with n vertices iff n is a power of 2. Corona product of two graphs has no LPST whenever the first graph has atleast two vertices. However, the corona product of any cocktail party graph $(\overline{nK_2})$ with a single vertex graph admits LPGST. In this regard, to check the existence of LPST on corona product of two graphs whenever the first graph has atleast 3 or more vertices. Also, finding the values of n for which the graph $k_n o \overline{k_m}$ exhibits LPGST for all m.

Keywords: Continuous time quantum walk · Laplacian perfect state transfer · Laplacian pretty good state transfer · Corona product.

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Cycles in 3-connected Planar Graphs

Chitrak Das

Ramakrishna Mission Vivekananda Educational And Research Institute

1 Abstract

Moon and Moser conjectured in 1963 that if G is a 3-connected planar graph on n vertices, then G contains a cycle of length at least $\Omega(n^{\log_3 2})$. This conjecture was later proven by Chen and Yu in their paper 'Long Cycles in 3-Connected Graphs' [1]. The results presented by Chen and Yu can be leveraged to address problems such as determining the minimum number of edge-disjoint cycles of length at least $\Omega(n^{\log_3 2})$ guaranteed in a 3-connected planar graph. This study involves investigating related problems in this domain.

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Gathering on a Continuous Circle by Autonomous Robot Swarm with defected view

Avisek Sharma

Department of Mathematics, Jadavpur University, Kolkata India

Abstract. A swarm of autonomous robots (mobile computing units) is deployed on a continuous circular path. This presentation focuses around gathering problem which asks the robots to gather at a point which is not decided beforehand. The robot movement is restricted on the circle. The robots are anonymous (no unique identifier), identical (physically indistinguishable), homogeneous (all robots execute the same algorithm). The robots operate in Look-Compute-Move cycle. In the "look" phase a robot finds out the position of the other visible robots. Taking this positions as input, in the "compute" phase the robot run an inbuilt algorithm and obtains a position. Finally in the "move" phase the robot moves the position. The gathering problem in this setting has been already studied in [1–3] all of has considered limited visibility for the robots. In [4], authors introduced another model called "defected view". In a (N, k)-defected view model, a robot can see k robots among other $N-1$ robots. This is a relevant model in the practical view point as in an erroneous environment a robot can miss out some other robots presence. In [1, 2], authors considered π visibility model where a robot cannot see the any robot situating at the angular distance π. In other robots, the farthest position on the circle from a robot is invisible for the robot. In the proposed work we want to generalised the model by introducing defected view model in it. In this work, we want to consider $(N, N-2)$-defected view model, i.e., a robot cannot see at most one robot which is farthest from it. This work will investigate what minimal capabilities are required for the robot to solve the gathering problem in $(N, N-2)$ model.

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Disjoint Dominating and 2-Dominating Sets in Graphs: Hardness and Approximation results

Soumyashree Rana[1], Sounaka Mishra[2], and Bhawani Sankar Panda[1]

[1] Department of Mathematics, Indian Institute of Technology Delhi, New Delhi, 110016, India
[2] Department of Mathematics, Indian Institute of Technology Madras, Chennai, 600036, India

Abstract. A set $D \subseteq V$ of a graph $G = (V, E)$ is a dominating set of G if each vertex $v \in V \setminus D$ is adjacent to at least one vertex in D, whereas a set $D_2 \subseteq V$ is a 2-dominating (double dominating) set of G if each vertex $v \in V \setminus D_2$ is adjacent to at least two vertices in D_2. A graph G is a DD_2-graph if there exists a pair (D, D_2) of dominating set and 2-dominating set of G which are disjoint. In this paper, we solve some open problems posed by M.Miotk, J. Topp and P.Żyliński (Disjoint dominating and 2-dominating sets in graphs, Discrete Optimization, 35:100553, 2020) by giving approximation algorithms for the problem of determining a minimal spanning DD_2-graph of minimum size (Min-DD_2) with an approximation ratio of 3; a minimal spanning DD_2-graph of maximum size (Max-DD_2) with an approximation ratio of 3; and for the problem of adding minimum number of edges to a graph G to make it a DD_2-graph (Min-to-DD_2) with an $O(\log n)$ approximation ratio. Furthermore, we prove that Min-DD_2 and Max-DD_2 are APX-complete for graphs with maximum degree 4. We also show that Min-DD_2 and Max-DD_2 are approximable within a factor of 1.8 and 1.5 respectively, for any 3-regular graph. Finally, we show the inapproximability result of Max-Min-to-DD_2 for bipartite graphs, that this problem can not be approximated within $n^{\frac{1}{6}-\varepsilon}$ for any $\varepsilon > 0$, unless P=NP.

Keywords: Domination· double domination· NP-complete· Approximation algorithm· APX-complete.

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Heterochromatic Geometric Transversals of Convex sets

Presenter: Soumi Nandi

Indian Statistical Institute

Abstract. An infinite family $\mathcal{F}$ of closed convex bodies in $\mathbb{R}^d$ has $(\aleph_0, q)$-property with respect to k-transversals if any countable subcollection of $\mathcal{F}$ contains q sets that can be pierced by a single k-flat (k-dimensional affine space). Observe that $(\aleph_0, q)$-property is a relaxation of the well-known (p, q)-property of Hadwiger and Debrunner (Archiv der Mathematik 1957) by replacing p with $\aleph_0$. Keller and Perles (Symposium on Computational Geometry 2022) introduced $(\aleph_0, q)$-property and they showed that if $\mathcal{F}$ is a collection of *fat* convex sets and satisfy $(\aleph_0, k+2)$-property with respect to k-transversals then $\mathcal{F}$ can be pierced by finitely many k-flats. I will present *countably colorful* generalizations of the above result and also establish their tightness by proving a number of *no-go* theorems.

Keywords: (p, q)-theorem · geometric transversals · convexity.

1 Open problem

Can we establish the same result without the *fatness* assumption?

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Locating and neighbor-locating colorings of graphs

Supraja D K

Indian Institute of Technology Dharwad, India
dksupraja95@gmail.com

Abstract. A proper k-vertex-coloring of a graph G is a *neighbor-locating k-coloring* if for each pair of vertices in the same color class, the sets of colors found in their neighborhoods are different. The neighbor-locating chromatic number $\chi_{NL}(G)$ is the minimum k for which G admits a neighbor-locating k-coloring. A proper k-vertex-coloring of a graph G is a *locating k-coloring* if for each pair of vertices x and y in the same color-class, there exists a color class S_i such that $d(x, S_i) \neq d(y, S_i)$. The locating chromatic number $\chi_L(G)$ is the minimum k for which G admits a locating k-coloring. It follows that $\chi(G) \leq \chi_L(G) \leq \chi_{NL}(G)$ for any graph G, where $\chi(G)$ is the usual chromatic number of G.
It is shown that for any three integers p, q, r with $2 \leq p \leq q \leq r$ (except when $2 = p = q < r$), there exists a connected graph $G_{p,q,r}$ with $\chi(G_{p,q,r}) = p$, $\chi_L(G_{p,q,r}) = q$ and $\chi_{NL}(G_{p,q,r}) = r$ [2]. It is interesting to know under what conditions does a graph have neighbor-locating number equal to locating chromatic number (resp. chromatic number).
Alcon *et al.* [1] proposed the following conjecture for Mycielski graph $\mu(G)$ of a graph G: For every graph G, $\chi_{NL}(\mu(G)) = \chi_{NL}(G) + 1$.
Till date, the conjecture is proved for complete multipartite graphs [1], paths, cycles, fans, wheels and comb graphs [3]. The conjecture still remains open.

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The complex problem of the complexity of Minimum Dominating set algorithms

Evangeline Prathibha M.

Department of Mathematics, Lady Doak College, Madurai. Tamil Nadu. India

Domination, a fundamental concept in graph theory, seeks to control a network by strategically positioning a minimal set of ”domineering” elements. Finding these sets efficiently has fueled research for decades, leading to a rich tapestry of algorithmic strategies. For many domination problems, NP-completeness casts a shadow of computational hardness. However, the battle is not lost. Parametrized complexity analysis is offering nuanced insights into the problem's inherent difficulty, paving the way for effective fixed-parameter tractable algorithms for specific problem configurations. Additionally, approximation algorithms and heuristic approaches are providing practical solutions for large-scale networks.

The classical bound for finding the minimum dominating set S is at $O(1.5137^n)$. This was discovered by van Rooij, J. M. M.; Nederlof, J.; van Dijk, T. C. in 2009. It was mentioned in their paper titled ”Inclusion/Exclusion Meets Measure and Conquer: Exact Algorithms for Counting Dominating Sets”, Proc. 17th Annual European Symposium on Algorithms, ESA 2009, Lecture Notes in Computer Science, vol. 5757, Springer, pp. 554565.

There is ongoing attempts in various fields, such as quantum computing, biomolecular computing to create MDS algorithms of better complexity. There is also domain specific attempts designed for social networks, sensor networks etc. There is no denying that MDS algorithms are NP-hard and that it is difficult to find a simple solution. The question that I ponder, is “What approach has actually yielded the best returns to create an MDS algorithm of improved complexity? And while we keep challenging the bound, is there a certain bound beyond which it is impossible to improve?”

Arbitrary Pattern Formation on Infinite Grid in presence of Crash Fault

Pritam Goswami

Jadavpur University, Kolkata-700032,India
pritamgoswami.math.rs@jadavpuruniversity.in

Abstract. The field of swarm robot algorithms extensively explores the Arbitrary Pattern Formation ($\mathcal{APF}$) problem. In this problem, a group of n robots is initially placed in an environment and are given a particular but arbitrary pattern defined by a set of points in a global coordinate system as input. The task is for the robots to navigate in a manner that concludes with them forming the specified pattern before termination.
Bose et al. initially presented this problem on an infinite grid ([1]), and subsequent studies have explored it, focusing on optimizing factors such as time, movement, and space requirements (Ghosh et al. [IJPEDS(J), 2023], Hector et al. [IPDPS, 2022], Sharma et al.[ICDCN,2024]). However, none of these works have addressed the potential scenario of robots encountering crashes.
In our work, we introduce the $\mathcal{APF}_{(n,f)}$ problem on an infinite rectangular grid, where n robots are initially placed on grid vertices, and up to f robots may crash. The objective is for the remaining robots to form the given pattern, incorporating the positions of crashed robots into the final pattern representation. Robots can move only to adjacent vertices and are not visible on edges.
Our findings reveal that, employing the $\mathcal{OBLOT}$ robot model (robots without persistent memory and lacking explicit communication), it is unfeasible to solve $\mathcal{APF}_{(n,1)}$ on an infinite rectangular grid using either a semi-synchronous or asynchronous scheduler. This leads to two important conclusions:

- To solve $\mathcal{APF}_{(n,1)}$ on an infinite rectangular grid with $\mathcal{OBLOT}$ robot model, it is necessary to have a fully synchronous scheduler.
- To solve $\mathcal{APF}_{(n,1)}$ on an infinite rectangular grid under semi-synchronous or, asynchronous scheduler it is necessary for the robots to have either finite persistent memory or explicit communication using finite size messages.

While the necessity aspects have been addressed, the sufficiency parts remain open challenges awaiting resolution.

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Injective Coloring of Interval Graphs

Rumki Ghosh

Department of Mathematics, Indian Institute of Technology Delhi

Abstract. An injective k-coloring of a graph $G = (V, E)$ is a function $f : V \to \{1, 2, \ldots, k\}$ such that for every pair of vertices u, v having a common neighbor, $f(u) \neq f(v)$. The injective chromatic number $\chi_i(G)$ of a graph G is the minimum value of k for which G admits an injective k-coloring. Given a graph G and a positive integer k, the Decide Injective Coloring Problem is to decide whether G admits an injective k-coloring. Decide Injective Coloring Problem is known to be NP-complete for chordal graphs. In this paper, we prove that the injective chromatic number of an interval graph is either $\Delta(G)$ or $\Delta(G) + 1$, where $\Delta(G)$ is the maximum degree of G. We also characterize the interval graphs having $\chi_i(G) = \Delta(G)$ and $\chi_i(G) = \Delta(G) + 1$. Finally, we present a linear time algorithm to find the injective chromatic number of an interval graph.

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Power Domination in Buckminsterfullerene

Ansily Silvaster
Department of Applied Mathematics & Computational Sciences,
PSG College of Technology, Coimbatore
aniesilvaster1998@gmail.com

A nonempty subset S of vertices of a graph $G = (V, E)$ is said to be a power dominating set (PDS) if every vertex and every edge in the graph are monitored by S. The power domination problem is NP-Complete in general. The power domination number $\gamma_p(G)$ is the minimum cardinality of a PDS of G. Any dominating set of G is a power dominating set G and therefore, $1 \leq \gamma_p(G) \leq \gamma(G)$.
Graph invariants are useful in studying certain properties of chemical compounds. Fullerenes, a highly symmetrical allotrope of carbon atoms joined by single and double bonds. The graph of Buckminsterfullerene C_{60} is a 60-vertex 3-regular 3-connected planar graph comprised of pentagonal and hexagonal faces. We observe that the power domination number is at most 4. There are 158 non-isomorphic Kekule structures (double bonds) of C_{60} [2]. These are equivalent to the distinct perfect matchings of C_{60} graph. Each double bond between the carbon atoms of C_{60} are compressed to a single vertex (edge compression) which results in a 4-regular graph H with 30 vertices and 60 edges. Joela [1] refers such compressed structure of a molecule as a submolecule of its double bond. The power domination of these 158 submolecules of C_{60} are studied and we found that $3 \leq \gamma_p(H) \leq 6$. The future work is to study the structural aspects of these submolecules and the contribution of vertices in attaining its power domination number.

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