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Indo-Slovenia Pre-Conference School on Algorithms and Combinatorics

February 12 - 13, 2024

Outline of Lectures

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Preface

Discrete Mathematics is a branch of mathematics involving discrete and distinct mathematical objects. It is viewed as one of the fundamental fields to understand the mathematical grounds of Computer Science, quintessentially Algorithms, Cryptography, Graph Theory, Computational Geometry, and related disciplines. Discrete Mathematics concerns itself with problems of (i) finding an optimal/extremal object and (ii) Combinatorics or the mathematics of counting the number of objects satisfying a set of properties among a family of discrete objects. Although the above-mentioned fields have origins centuries before, these fields are evergreen and prove to be highly promising due to immense research conducted around the globe.

The Indo-Slovenia Pre-conference School is an initiative to bring together eminent experts in the fields of Discrete Mathematics to provide a forum for researchers to discuss the current advancements in Algorithms and Discrete Applied Mathematics. The knowledge dissemination and sharing will encourage inquisitive students to become prospective researchers. The school intends to improve research through collaboration with researchers worldwide. The school is aimed at fulfilling two purposes: (i) as a Pre-Conference School for CALDAM 2023, and (ii) as an Indo-Slovenia School on Algorithms and Combinatorics. The school is organized by Indian Institute of Technology Bhilai, Chhattisgarh, India.

CALDAM Indo-Slovenia Pre-Conference School on Algorithms and Combinatorics

February 12-13, 2024

February 12, 2024		February 13, 2024	
08:30 - 09:15	Registration	-	-
09:15 - 09:30	Inaugural Session	-	-
09:30 - 10:30	**Technical Session – 1** Iztok Peterin Topic: On Properties of Modular and Direct-Co-Direct Products	09:30 - 10:30	**Technical Session – 5** Vesna Iršič Topic: Cops and Robber on Surfaces of Constant Curvature
10:30-11:00	Tea break		
11:00 - 12:00	**Technical Session – 2** Swagato Sanyal Topic: A Tutorial on Communication Complexity and its applications	11:00 - 12:00	**Technical Session – 6** Ragesh Jaiswal Topic: Some Results on Outlier and Constrained Clustering
12:00 - 13:00	Bodhayan Roy Topic: Conflict-Free Coloring of Polygons	12:00 - 13:00	Akanksha Agrawal Topic: Hybrid Parameterizations for Graph Problems
13:00 - 14:30	Lunch Break		
14:30 - 15:30	**Technical Session – 3** Riste Skrekoviski Topic: Selected Topics on Wiener Index	14:30 - 15:30	**Technical Session – 7** Tanja Dravec Topic: Grundy Domination Invariants
15:30 - 16:00	Tea Break		
16:00 - 18:30	**Technical Session – 4** Young Researchers' Forum (YRF 2024)	16:00 - 17:00	**Technical Session – 8** I. Vinod Reddy Topic: On Conflict-Free Coloring of Graphs
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On Properties of Modular and Direct-Co-Direct Products

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Any product of two graphs G and H has a vertex set $V(G) \times V(H)$. For edges, we consider three objects in every factor for the definition of an edge in a product: either a vertex or an edge or a non-edge. Combining the mention objects from one factor with the same objects in the other factor gives eight different possibilities (notice that vertex by vertex must be ignored to avoid loops) to have an edge or a non-edge in a product. So, all together $2^8 = 256$ different graph products exists, see Imrich [4] or an exhaustive monograph on graph products by Hamack et al. [3].

There is a variety of different products from not really interesting ones like empty or complete product, where we have no or all edges, respectively, to more interesting ones. Over the years four of them—Cartesian $G\Box H$, direct $G \times H$, strong $G \boxtimes H$ and lexicographic $G \circ H$—gain a special status and we call them standard products. Two vertices (g, h) and (g', h') of a product are adjacent by a Cartesian edge when $g = g'$ and $hh' \in E(H)$ or $gg' \in E(G)$ and $h = h'$. Similar, (g, h) and (g', h') are adjacent by a direct edge when $gg' \in E(G)$ and $hh' \in E(H)$. Now, edge set of Cartesian product $E(G\Box H)$ contains exactly all the Cartesian edges, while the edge set of direct product $E(G \times H)$ contains exactly all the direct edges. For strong product we have $E(G \boxtimes H) = E(G\Box H) \cup E(G \times H)$. Vertices (g, h) and (g', h') are adjacent in $G \circ H$ when $gg' \in E(G)$ or $g = g'$ and $hh' \in E(H)$.

All four of standard products are associative, but the lexicographic is not commutative due to non-symmetric definition of $E(G \circ H)$. Among all graph products only 20 are associative, see [3, 4], together with the mentioned four standard products. Further only ten of the mentioned 20 products are also commutative. We can join them into pairs of a product $G * H$ together with its complementary product $G\overline{*}H \stackrel{\text{def}}{=} \overline{\overline{G} * \overline{H}}$ where $\overline{G}$ denotes the complement of a graph G. So, six of them represent either a standard product or its complementary product, next two are empty and complete product and finally modular product $G \diamond H$, where $E(G \diamond H) = E(G\Box H) \cup E(G \times H) \cup E(\overline{G} \times \overline{H})$, and its complementary product. However, almost all the attention in the literature is devoted to the mentioned four standard products. One can find some exceptions, like [4] where modular product was investigated and Nowakowsky and Rall [9] from the point of multiplicity of some invariants like independent, chromatic and domination numbers. Therefore one could also use term "the forgotten (associative and commutative) product" for modular product.

The situation is even worst in the case of other non-commutative products (beside lexicographic product) or even non-associative products. Here we consider a direct-co-direct product (or DcD product for short) $G \circledast H$ with edge set $E(G \circledast H) = E(G \times H) \cup E(\overline{G} \times \overline{H})$. It is easy to see that $G \circledast H$ is commutative, but it is not associative.

The study of graph products can be divided into two main directions. First is a structural one, where we are interested whether a (big) graph G is a certain product at all, and if so, can we derive its factors and are this factors unique. This was solved for Cartesian, strong and direct product in affirmative for some graph classes and polynomial algorithms were presented for the mentioned factorizations, see [5] and the references there in.

The other approach is to derive properties of a product from properties of its factors. The benefit of this approach is that the factors are often considerably smaller than the product which results in a faster algorithmic approach. In recent decades numerous results of this type were presented in the literature. Let us mention here only Hedetniemi's conjecture for the chromatic number of the direct product $\chi(G \times H) = \min\{\chi(G), \chi(H)\}$ that was recently disproved by Shitov [10] and Vizing's conjecture for the domination number of Cartesian product $\gamma(G\Box H) \leq \gamma(G)\gamma(H)$ which is still open, see the last survey by Brešar et al. [2]. Both of these two conjectures inspired many deep results. We follow the later approach and present several recently discovered properties of modular and direct-co-direct products with the respect of some properties of their factors. Some properties are straightforward for the mentioned two products as we will see for the independence number of modular product.

On the other hand, one can expect many interesting results for many graph parameters and also some really difficult ones like mentioned Hedetniemi's and Vizing's conjecture. In particular we present:

- distance formula for modular product, see [8];
- distance formula for DcD product in the case of connected factors, see [6];
- distance formula for DcD product in the case of disconnected factors, see [7];
- strong metric dimension for modular product, see [8];
- domination number of modular product, see [1].

Beside that several open problems will be presented among the way.

Keywords: modular product, direct-co-direct product, distance formula, strong metric dimension, domination number

AMS Subject Classification: 05C76, 05C12, 05C69

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Cops and Robber on Surfaces of Constant Curvature

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Abstract

In 2021, Mohar [3] introduced the game of Cops and Robber on geodesic spaces. The game captures the behavior of the Cops and Robber game played on graphs and that of continuous pursuit-evasion games. One of the open problems for the Cops and Robber game on graphs concerns the upper bound on the cop number of graphs embeddable in a surface of genus g. The upper bound is known to be $O(g)$, but it is conjectured that the bound is in fact $O(\sqrt{g})$. An analogous bound is believed to hold for geodesic surfaces as well. In [4] Mohar conjectured that the cop number of a geodesic surface of genus g is at most $O(\sqrt{g})$. Surprisingly, this upper bound can be significantly improved on surfaces of constant curvature which will be the main focus of this talk.

It turns out that the cop number of compact spherical and euclidean surfaces is at most 2 [1]. Even more surprisingly, the cop number of compact hyperbolic surfaces is also at most 2, independently of their genus [2]. We will also consider the strong cop number of these surfaces and present several generalizations to higher-dimensions.

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Grundy Domination Invariants

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Abstract

The main topic of this lecture is domination theory. We will present and study four related concepts Grundy domination, Grundy total domination, Z-Grundy domination and L-Grundy domination. In a graph $G = (V, E)$ a sequence $S = (v_1, \ldots, v_k)$ of distinct vertices of G is a closed (open) neighborhood sequence if for each $i \in \{2, \ldots, k\}$,

$$N[v_i] \setminus \bigcup_{j=1}^{i-1} N[v_j] \neq \emptyset \tag{1}$$

$(N(v_i) \setminus \bigcup_{j=1}^{i-1} N(v_j) \neq \emptyset)$. The maximum length of a closed (open) neighborhood sequence in G is called Grundy domination number (Grundy total domination number) of G [6, 7]. Z-Grundy and L-Grundy domination number of a graph G are defined in a similar way by changing exactly one of closed neighborhods in (1) by open neighborhood [2].

In the lecture we present some recent results on the four types of Grundy domination invariants. We first present relations between all four Grundy numbers and (total) domination number, i.e. we show that for any connected graph G not isomorphic to complete graph the following holds:

(a) $\gamma(G) \leq \gamma_{\rm gr}^{\rm Z}(G) \leq \gamma_{\rm gr}(G) \leq \gamma_{\rm gr}^{L}(G) - 1$ [2],
(b) $\gamma_t(G) \leq \gamma_{\rm gr}^{\rm Z}(G) \leq \gamma_{\rm gr}^{t}(G) \leq \gamma_{\rm gr}^{L}(G)$ [2, 3].

Graphs G with $\gamma_t(G) = \gamma_{\rm gr}^t(G)$ were studied in [1, 7, 10], but just partial results are known. For example graphs with $\gamma_t(G) = \gamma_{\rm gr}^t(G) = 4$ and graphs with $\gamma_t(G) = \gamma_{\rm gr}^t(G) = 6$ are characterized inside bipartite graphs and it was proved that there are no chordal graphs with $\gamma_t(G) = \gamma_{\rm gr}^t(G)$ and no graphs with $\gamma_t(G) = \gamma_{\rm gr}^t(G) = 2\ell + 1$ for any $\ell \in \mathbb{N}$ [10, 1].

Problem 1 *Characterize graphs with* $\gamma_t(G) = \gamma_{\rm gr}^t(G) = 4$.

Since the family of non-complete graphs satisfying $\gamma_t(G) = \gamma_{\rm gr}^t(G)$ is a subfamily of a family of graphs having $\gamma_t(G) = \gamma_{\rm gr}^{\rm Z}(G)$, we cane pose the following question.

Question 1 *Is there a connected chordal graph* G *with* $\gamma_t(G) = \gamma_{\rm gr}^{\rm Z}(G) = k$ *for* $k \geq 4$*?*

There are no such graphs for $k = 3$ [3].

On the other hand, the upper total domination number of a graph G, $\Gamma_t(G)$, (the maximum cardinality of a minimal total dominating set) is not bounded above by the Z-Grundy domination number of G. We have proved the following.

Theorem 1 *[3] If* G *is isolate-free graph, then* $\Gamma_t(G) \leq 2\gamma_{\rm gr}^{\rm Z}(G)$.

Problem 2 *Characterize graphs* G *with* $\Gamma_t(G) = 2\gamma_{\rm gr}^{\rm Z}(G)$.

There exist trivial upper bounds with respect to the order and the minimum degree of a graph G for all four Grundy domination numbers, i.e. $\gamma_{\rm gr}^{\rm Z}(G) \leq \gamma_{\rm gr}(G) \leq |V(G)| - \delta(G)$, $\gamma_{\rm gr}^t(G) \leq \gamma_{\rm gr}^L(G) \leq |V(G)| - \delta(G) + 1$ [2, 6, 7, 11]. We are again interested in the extremal graphs, i.e. graphs that attain those bounds. We connect the graphs satisfying $\gamma_{\rm gr}^{\rm Z}(G) = |V(G)| - \delta(G)$ with graphs having power domination number equal to 1 and then prove that graphs with $\gamma_{\rm gr}^{\rm Z}(G) = |V(G)| - \delta(G)$ are exactly graphs, whose vertices can be partitioned into $\delta(G)$ internally disjoint induced paths with one common end-vertex and some special properties about edges between those paths [3]. Extremal graphs for the other three Grundy invariants are not known.

Problem 3 *Characterize graphs* G *satisfying* $\gamma_{\rm gr}(G) = |V(G)| - \delta(G)$ *(*$\gamma_{\rm gr}^t(G) = |V(G)| - \delta(G) + 1$ *or* $\gamma_{\rm gr}^L(G) = |V(G)| - \delta(G) + 1$*).*

For L-Grundy domination some partial results for graphs with minimum degree 1 were already presented [11].

In the talk we also present complexity results. We present NP-completeness of the decision versions for all four Grundy domination numbers of a graph. It is known that the decision versions of the Grundy domination number, Grundy total domination number and L-Grundy domination number are NP-complete even when restricted to bipartite graphs [2, 7, 9].

Question 2 *Let G be an arbitrary bipartite graph. Can Z-Grundy domination number of G be computed in polynomial time?*

For each of the four invariants we present some classes of graphs in which the problem is polynomial. There exists a linear time algorithm for Grundy domination number of a tree, but no explicit formula is known [6]. On the other hand, if T is a tree, then $\gamma_{\rm gr}^{L}(T) = |V(T)|$ [4], $\gamma_{\rm gr}^{t}(T) = 2\beta(T)$, where $\beta(G)$ denotes vertex cover number of a graph G [8] and $\gamma_{\rm gr}^{Z}(T) = P(T)$, where $P(G)$ denotes path cover number of G [12].

A graph G is a split graph, if its vertices can be partitioned into two subsets, where one subset is a clique and the other is an independent set. All four Grundy invariants were investigated also in split graphs. Grundy domination number of a split graph can be computed in polynomial time [6], while L-Grundy and Grundy total domination number problems are NP-complete [4, 8].

Question 3 *Is there a polynomial time algorithm that returns Z-Grundy domination number of an arbitrary split graph?*

Grundy domination number was studied also in interval graphs [5]. Linear time algorithm that computes Grundy domination number of an interval graph was presented. The key argument of the algorithm was nice behaviour of Grundy domination number, when a simplicial vertex or a twin vertex is deleted from the graph. Since Z-Grundy domination number behaves similarly, we finish with the following problem.

Problem 4 *Find an efficient algorithm that returns a Z-Grundy dominating sequence of an interval graph.*

Keywords: Grundy domination number, Grundy total domination number, Z-Grundy domination number, L-Grundy domination number, zero forcing, total domination, upper total domination, split graph, interval graph

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Conflict-free Colouring of Polygons

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In general, conflict-free colouring of a graph is assigning colours to vertices of that graph such that the neighborhood of each vertex contains at least one unique colour. This problem was first studied by Biggs with the name *perfect code*, which is essentially conflict-free colouring of a graph using only one colour [4, 10]. Later on, this topic aroused interest on polygon visibility graphs when the field of robotics became widespread [7, 8]. Consider a scenario where a mobile robot traverses a room from one point to another, communicating with the wireless sensors placed on the corners of the room. Even if the robot has full access to the map of the room, it cannot determine its location precisely because of accumulating rounding errors. And thus it needs clear markings in the room to guide itself to the end point in an energy efficient way. To guide a mobile robot with wireless sensors, two properties must be satisfied. First one is, no matter where the robot is in the polygon, it should hear from at least one sensor. That is, the placed sensors must together *guard* the whole room and leave no place uncovered. The second one is, if the robot hears from several sensors, there must be at least one sensor broadcasting with a frequency that is not reused by some other sensor in the range. That is, the sensors must have *conflict-free* frequencies. If these two properties are satisfied, then the robot can guide itself using the deployed wireless sensors as landmarks. This problem is also closely related to frequency assignment problem in wireless networks [1, 3]. One can easily solve this problem by placing a sensor at each corner of the room, and assigning a different frequency to each sensor. However, this method becomes very expensive as the number of sensors grow [1, 11]. Therefore, the main goal in this problem is minimize the number of different frequencies assigned to sensors. Since the cost of a sensor is comparatively very low, we do not aim to minimize the number of sensors used.

The above scenario is geometrically modeled as follows. The room is a simple polygon with n vertices. There are m sensors placed in the polygon (usually on some of its vertices), and two different sensors are given two different colours if, and only if they broadcast in different frequencies. We consider simple polygons (informally, "without holes"), usually non-convex. Two points p_1 and p_2 of a polygon P are said to *see* each other, or be *visible* to each other, if the line segment $\overline{p_1p_2}$ fully belongs to P. In this context, we say that a guard g *guards* a point x of P if the line segment $\overline{gx}$ fully belongs to P. A polygon P is a *weak visibility polygon* if P has an edge uv such that for every point p of P there is a point p' on uv seeing p. A solution of *conflict-free chromatic guarding* of a polygon P consists of a set of *guards* in P, and an assignment of colours to the guards (one colour per guard) such that the following holds; every *viewer* v in P (where v can be any point of P in our case) can see a guard of colour c such that no other guard seen by v has the same colour c. In the *point-to-point* (P2P) variant the guards can be placed in any points of P, while in the *vertex-to-point* (V2P) variant the guards can be placed only at the vertices of P. In the V2V variant viewers are also restricted to the vertices. In all variants the goal is to minimize the number of colours (e.g., frequencies) used.

The aforementioned P2P conflict-free chromatic guarding (art gallery) problem has been studied in several papers. Bärtschi and Suri gave an upper bound of $O(\log^2 n)$ colours on simple n-vertex polygons [3]. Later, Bärtschi et al. improved this upper bound to $O(\log n)$ on simple polygons [2], and Hoffmann et al. [9], while studying the orthogonal variant of the problem, have given the first nontrivial lower bound of $\Omega\big(\log\log n/\log\log\log n\big)$ colours holding also in the general case of simple polygons. In the V2P variant of the problem, guards should be placed on polygon vertices and viewers can be any points of the polygon. Note that there are some fundamental differences between point and vertex guards, e.g., funnel polygons can always be guarded by one point guard (of one colour) but they may require up to $\Omega(\log n)$ colours in the V2P conflict-free chromatic guarding, as shown in [2]. Hence, extending a general upper bound of $O(\log n)$ colours for point guards on simple polygons by Bärtschi et al. [2] to the more restrictive vertex guards is a challenge, which Çagirici et al. [5, 6] approached with an $O(\log^2 n)$ bound. The same bound of $O(\log^2 n)$ colours was attained earlier by [2] when allowing multiple guards at the same vertex. Çagirici et al. also gave a polynomial-time algorithm to find the optimum number m of vertex-guards to guard all the points of a funnel, and showed that the number of colours in the corresponding conflict-free chromatic guarding problem is $\log m + \Theta(1)$ [5, 6]. This leads to an approximation algorithm for V2P conflict-free chromatic guarding of a funnel, with only a constant (+4) additive error. Finally they showed that the V2V version of the problem is NP-complete for simple polygons. In this talk we will discuss some of the above results, and state some related unsolved problems.

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On Conflict-free Coloring of Graphs

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A coloring of a graph is called conflict-free if every vertex has a uniquely colored vertex in its neighborhood. The conflict-free coloring problem is to color the vertices of a graph using the minimum number of colors such that the coloring is conflict-free. The conflict-free coloring problem was introduced by Even et al. [6] to study the frequency assignment problem for cellular networks. The problem is NP-complete on general graphs [7] and its complexity has been well studied on several restricted graph classes [1, 3, 4]. In the first part of this talk we survey the results on parameterized complexity of conflict-free coloring of graphs [2, 7, 10].

In the second part we look at a variant of conflict-free coloring called odd coloring. A proper vertex coloring of a graph G is odd, if for every non-isolated vertex v of a graph, the set $N(v)$ has a color that appears odd number of times. The problem was introduced recently by Petruševski and Škrekovski [9]. We survey some recent results on odd coloring of graphs [5, 8].

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Some results on outlier and constrained clustering

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Abstract. The talk will consist of two parts:

(Part I) In this work, we study a range of *constrained* versions of the *k-supplier* and *k-center* problems. In the classical (*unconstrained*) k-supplier problem, we are given a set of clients C in a metric space $\mathcal{X}$, with distance function $d(.,.)$. We are also given a set of feasible facility locations $L \subseteq \mathcal{X}$. The goal is to open a set F of k facilities in L to minimize the maximum distance of any client to the closest open facility, i.e., minimize, $\text{cost}(F, C) \equiv \max_{j \in C} \left\{ d(F, j) \right\}$, where $d(F, j)$ is the distance of client j to the closest facility in F. The k-center problem is a special case of the k-supplier problem where $L = C$. We study various constrained versions of the k-supplier problem such as: *capacitated*, *fault-tolerant*, ℓ-diversity, etc. These problems fall under a broad framework of *constrained clustering*. A unified framework for constrained clustering was proposed by [**?**] in the context of the k-median and k-means objectives. We extend this framework to the k-supplier and k-center objectives in this work. This unified framework allows us to obtain results simultaneously for the following constrained versions of the k-supplier problem: *r-gather, r-capacity, balanced, chromatic, fault-tolerant, strongly private, ℓ-diversity, and fair k-supplier problems, with and without outliers.* We design Fixed-Parameter Tractable (FPT) algorithms for these problems. FPT algorithms have polynomial running time if the parameter under consideration is a constant. This may be relevant even to a practitioner since the parameter k is a small number in many real clustering scenarios. We obtain the following results:

- We give 3 and 2 approximation algorithms for the constrained k-supplier and k-center problems, respectively, with FPT running time $k^{O(k)} \cdot n^{O(1)}$, where $n = |C \cup L|$. Moreover, these approximation guarantees are tight; that is, for any constant $\varepsilon > 0$, no algorithm can achieve $(3-\varepsilon)$ and $(2-\varepsilon)$ approximation guarantees for the constrained k-supplier and k-center problems in FPT time, assuming $\mathsf{FPT} \neq \mathsf{W}[2]$.
- We study the constrained clustering problem with outliers. Our algorithm gives 3 and 2 approximation guarantees for the constrained *outlier* k-supplier and k-center problems, respectively, with FPT running time $(k+m)^{O(k)} \cdot n^{O(1)}$, where $n = |C \cup L|$ and m is the number of outliers.
- Our techniques generalise for distance function $d(.,.)^z$. That is, for any positive real number z, if the cost of a client is defined by $d(.,.)^{1.15z}$ instead of $d(.,.)$, then our algorithm gives $3^{1.15z}$ and $2^{1.15z}$ approximation guarantees for the constrained k-supplier and k-center problems, respectively.

(*This part is based on a joint work with Dishant Goyal (IIT Delhi) and appeared in Theoretical Computer Science Journal.*)

(Part II) Constrained clustering problems generalize classical clustering formulations, e.g., k-median , k-means , by imposing additional constraints on the feasibility of a clustering. There has been significant recent progress in obtaining approximation algorithms for these problems, both in the metric and the Euclidean settings. However, the outlier version of these problems, where the solution is allowed to leave out m points from the clustering, is not well understood. In this work, we give a general framework for reducing the outlier version of a constrained k-median or k-means problem to the corresponding outlier-free version with only $(1+\varepsilon)$-loss in the approximation ratio. The reduction is obtained by mapping the original instance of the problem to $f(k, m, \varepsilon)$ instances of the outlier-free version, where $f(k, m, \varepsilon) = \left(\frac{k+m}{\varepsilon}\right)^{O(m)}$. As specific applications, we get the following results:

- First FPT (*in the parameters k and m*) $(1+\varepsilon)$-approximation algorithm for the outlier version of capacitated k-median and k-means in Euclidean spaces with *hard* capacities.

- First FPT (*in the parameters k and m*) $(3+\varepsilon)$ and $(9+\varepsilon)$ approximation algorithms for the outlier version of capacitated k-median and k-means , respectively, in general metric spaces with *hard* capacities.
- First FPT (*in the parameters k and m*) $(2-\delta)$-approximation algorithm for the outlier version of the k-median problem under the Ulam metric.

Our work generalizes the results of [**?**] and [**?**] to a larger class of constrained clustering problems. Further, our reduction works for arbitrary metric spaces and so can extend clustering algorithms for outlier-free versions in both Euclidean and arbitrary metric spaces.
(*This part is based on a joint work with Amit Kumar (IIT Delhi) and appeared in ISAAC 2023.*)

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A Tutorial on Communication Complexity and its Applications

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The two-party model of communication was introduced by Yao in 1979, and has since become remarkably useful as a general "lower-bound technique"; in theoretical computer science. The model quantifies the number of bits that two parties, who jointly hold an input to a computational task, need to exchange to carry out the task. The framework has been used to establish lower-bounds on the amount of resources necessary for a variety of computational problems and computational models: Data structures, space-time trade-off for Turing Machines, time-area trade-off for VLSI design, space lower-bounds for streaming algorithms, circuit lower bounds, combinatorial auction, etc. The tutorial will be an introduction to this fascinating area. We will start with the basic concepts of the model. We will then see some of its applications to computation. Along the way, we will see some open questions and possible directions for future research sanay

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Selected Topics on Wiener Index

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The Wiener index is defined as the sum of distances between all unordered pairs of vertices in a graph G, i.e.

$$W(G) = \sum_{\{u,v\} \subseteq V(G)} d(u, v).$$

The Wiener index, first introduced in 1947 by Harold Wiener, was originally designed to estimate the boiling points of paraffin. It has become one of the most renowned and extensively studied topological indices, and continues to be a vibrant area of research.

This talk presents some directions of research that are proposed in the survey

M. Knor, R.Š., A. Tepeh, *Selected topics on Wiener index*, arXiv:2303.11405, to appear in Ars Math. Contemp.

In this work, we have compiled answers to some questions, provided additional insights into the topic of extremal values of the Wiener index in various contexts, and introduced a range of intriguing problems and conjectures. More particularly, in the talk, we will focus on the following lines of research:

1. Minimum Wiener index for chemical graphs: Are the chemical graphs on $n \geq 5$ vertices that attain the minimum 4-regular graphs?
2. Regular graphs vs. diameter: In the realm of regular graphs could it be the length of the diameter related with the magnitude of the Wiener index.
3. Šoltés problem: Can we remove vertices in a graph without changing the value of its Wiener index?
4. Ratio of Wiener index of iterated line graphs: Assessing the ratio of growth of the Wiener index when applying the line-graph operation.
5. Wiener on digraphs: How can this concept be extended to directed graphs?

Hybrid Parameterizations for Graph Problems

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Parameterized Complexity is a framework that was originally designed to cope with NP-hardness. Parameterized Complexity can be thought of as a two-dimensional analogue of the classical complexity theory, where in addition to the input size, one studies how a secondary measure, called the parameter, that captures a certain property of the input or output (or both) affects the complexity of the problem.

Many graph problems are NP-hard and thus have been extensively studied in the realm of Parameterized Complexity. Most of the studies of graph problems have been for parameters like solution size or structural parameters like treewidth. In this talk we will revisit structural graph parameters like treewidth and tree depth, and then delve into the newly introduced hybrid parameters generalizing them. We will review some recent results on parameterized algorithms for these hybrid parameters and look at an interesting equivalence result of these parameters to the standard parameters

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