Cops and Robber on surfaces of constant curvature

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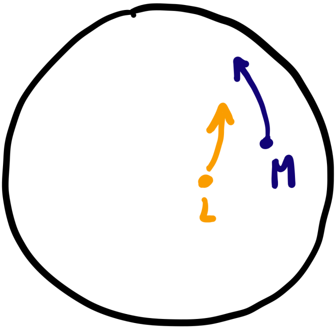
Pursuit-evasion games

Example (Rado 1930s, Littlewood 1953, 1986)

A lion and a man move in an arena with the same maximum speed. Can the lion catch the man in finite time?



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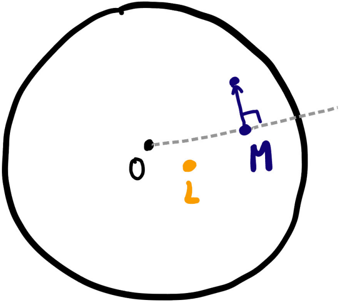
Besicovitch 1952: no



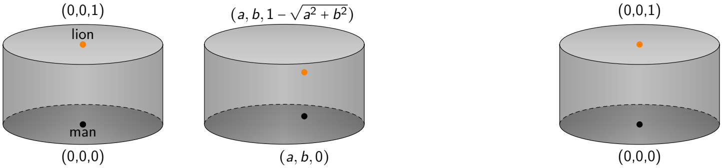
Pursuit-evasion games

Theorem (Bollobás, Leader, Walters, 2011)

There exists a compact metric space in which both players have winning strategies in the lion and man game.

Let D be a disk, $I = [0, 1]$. Let $S = D \times I$ and take ℓ_∞ metric on S.

- Lion keeps the disk coordinate the same as the man.
- The man moves away from lion's projectin and then uses Besicovitch's strategy.



The game of cops and robber on graphs

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The cops and robber occupy vertices of the graph. At the beginning of the game, robber chooses starting positions. On a turn, the cops or the robber can move to a neighboring vertex, or stay in the same position.

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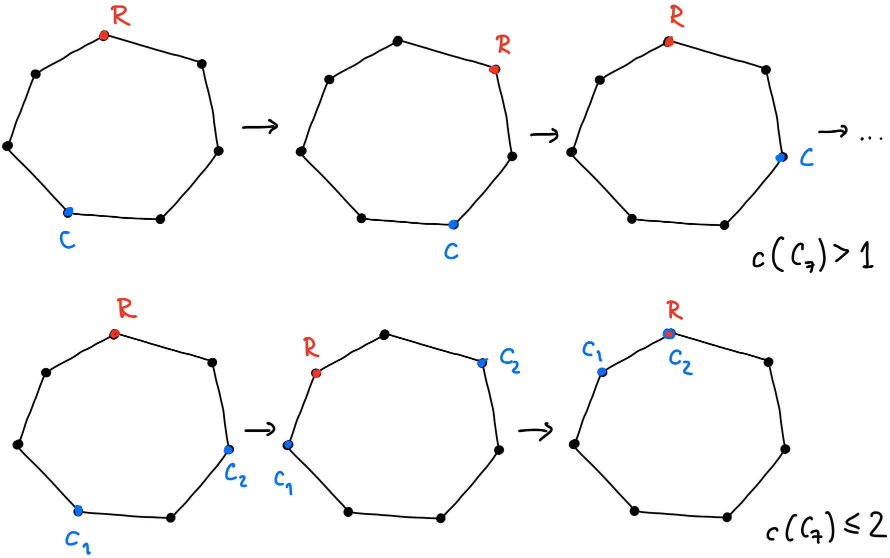
The cops win if after some finite number of rounds, one of them occupies the same vertex as the robber (capture). The robber wins if he can evade capture indefinitely.

The *cop number* $c(G)$ of a graph G is the minimum number k such that k cops can win the game on G.

The game of cops and robber on graphs



Cops and Robber on planar graphs

Theorem (Aigner, Fromme, 1984)

If G is a planar graph, then $c(G) \leq 3$.

Guarding shortest paths

An induced subgraph H of G is *k-guardable* if after finitely many moves, k cops can arrange themselves in H such that the robber is immediately captured upon entering H.

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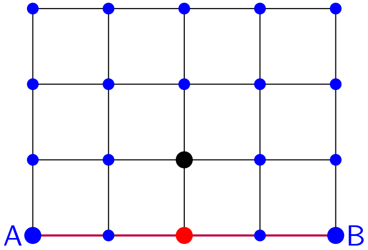
Isometric paths are 1-guardable.

c = cop's position, r = robber's position, P = shortest path from A to B



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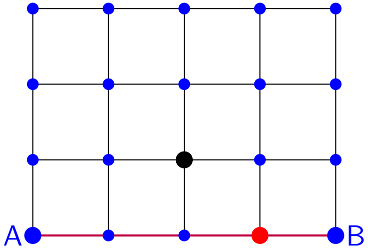

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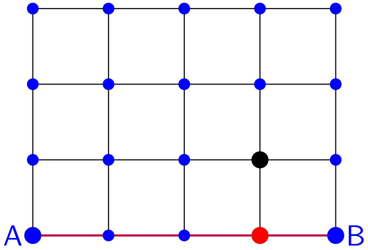

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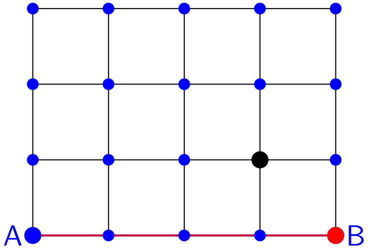

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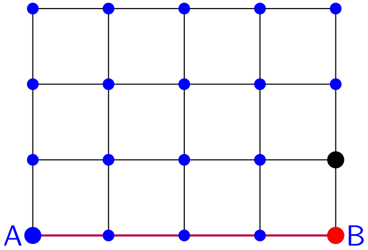

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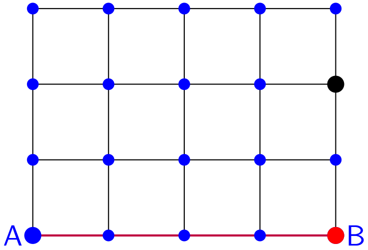

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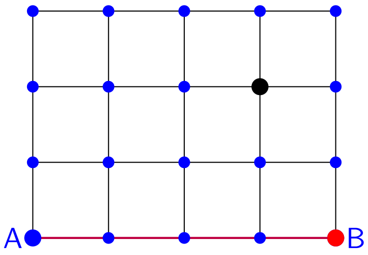

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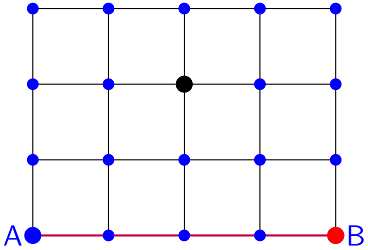

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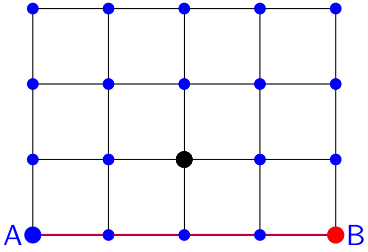

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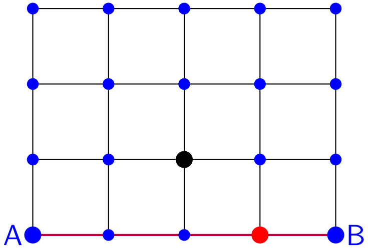

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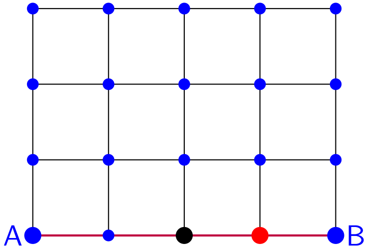

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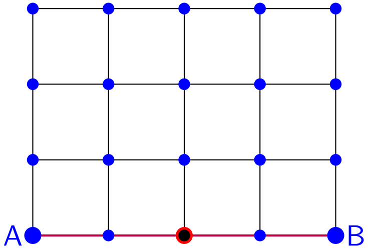
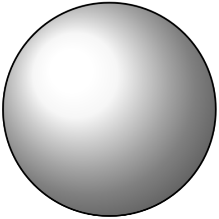
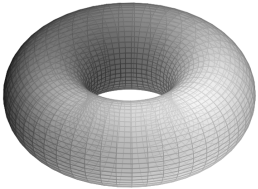


Surfaces

A *topological surface* is a connected topological space which is locally homeomorphic to an Euclidean disk.





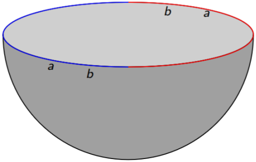


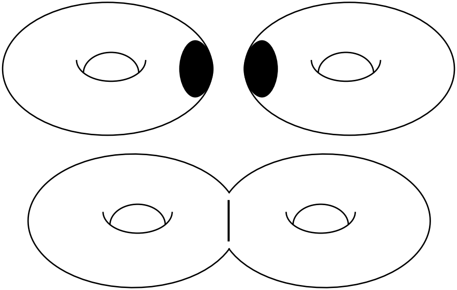
Figure: A sphere, a torus and a projective plane.

Classification theorem for surfaces

Every compact topological surface is homeomorphic to either

- the sphere ($g = 0$)
- the connected sum of g tori for $g \geq 1$,
- or the connected sum of $g + 1$ real projective planes for $g \geq 0$,

where g is the *genus* of the surface.

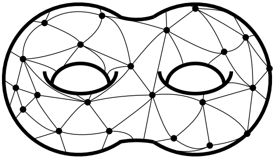


Graphs on surfaces

Conjecture (Mohar, 2009)

Let G be a graph embeddable in a surface of genus g. Then

$$c(G) \leq g^{\frac{1}{2}+o(1)}.$$



- $c(G) = O(g)$ (Quilliot, 1985; Andreae, 1986; Schroeder, 2001; Clarke, Fiorini, Joret, Theis, 2011; Erde, Lehner, Pitz, Bowler, 2019)

Meyniel's conjecture

Conjecture (Meyniel, 1985)

Let G be a graph on n vertices. Then

$$c(G) = O(n^{\frac{1}{2}}).$$

- Best known bound: $c(G) = O\left(\frac{n}{2^{1-o(1)}\sqrt{\log_2(n)}}\right)$ (Lu, Peng, 2012; Frieze, Krivelevich, Loh, 2012; Scott, Sudakov 2011)
- Weak Meyniel conjecture: $c(G) = O(n^{1-\varepsilon})$ for some $\varepsilon > 0$.

A new game?

Similar to

- the game of cops and robber on graphs, and
- differential pursuit-evasion games (man and lion),

but

- not played just on graphs, and
- played in discrete-time.

Geodesic spaces

(X, d) metric space

(x, y)-path is a continuous map $\gamma\colon [0,1] \to X$ where $\gamma(0) = x$ and $\gamma(1) = y$

Length $\ell(\gamma) = \sup_{0=t_0<t_1<\cdots<t_n=1} \sum_{i=1}^{n} d(\gamma(t_{i-1}), \gamma(t_i))$

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The metric space X is a *geodesic space* if for every $x, y \in X$ there is an (x, y)-path whose length is equal to $d(x, y)$.

Alternatively, equip any compact path-connected metric space X with the intrinsic metric (shortest path distance) to obtain a geodesic space.

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Examples: Euclidean space $\mathbb{R}^n$, n-dimensional sphere, Riemannian manifold, connected cell complex (including metric graphs).

The game of cops and robber on geodesic spaces

X a compact geodesic space with intrinsic metric d
The game of cops and robber on the game space X with k cops (introduced by Mohar 2021):

- the first player controls the robber R, positioned at $r \in X$
- the second player controls k cops $C_1, \ldots, C_k$, positioned at $c = (c_1, \ldots, c_k) \in X^k$

The robber selects an initial position for all players:
$Y_0 = (r^0, c_1^0, \ldots, c_k^0) \in X^{k+1}$.
Robber also selects an agility function $\tau \colon \mathbb{N} \to \mathbb{R}_+$, $\sum_{n \geq 1} \tau(n) = \infty$.

The game of cops and robber on geodesic spaces

After $n-1$ steps, the players are in position $(r^{n-1}, c_1^{n-1}, \dots, c_k^{n-1}) \in X^{k+1}$. In the nth step, each player can move with unit speed up to a distance at most $\tau(n)$ from its current position.

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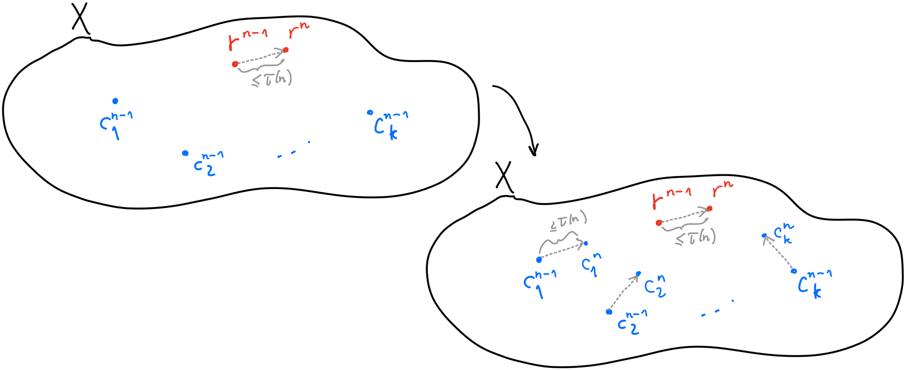
First, the robber moves to $r^n \in X$, $d(r^{n-1}, r^n) \le \tau(n)$. This is revealed to the cops. Then each cop moves to a new position c_i^n, $d(c_i^{n-1}, c_i^n) \le \tau(n)$.



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If the game never stops, then the value of the game is

$$v = \inf_{n \geq 0} \min_{i \in [k]} d(r^n, c_i^n).$$

If the value is 0, we say that the *cops win* the game; otherwise the *robber wins*. Note that the cops can win even if they never catch the robber.

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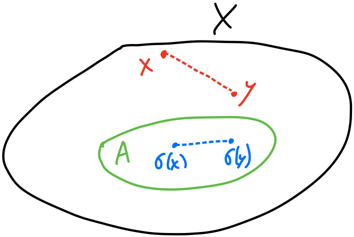
The minimum integer k such that k cops win the game on X is denoted by $c(X)$ and called the *cop number* of X. If such k does not exist, then we set $c(X) = \infty$. Similarly we define the *strong cop number* $c_0(X)$ as the minimum k such that k cops can always catch the robber.

Guarding

Let X be the game space and let $A \subseteq X$.
The *shadow* of the robber is $\sigma : X \to X$ such that $\sigma|_A = id|_A$ and $d(x, y) \geq d(\sigma(x), \sigma(y))$.



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If a cop comes to the point $\sigma(r)$, where r is the robber's position, then we say that the cop *caught* the shadow $\sigma(r)$ of the robber. It is easy to see that if after that robber ever enters A, then the cop can catch the robber. So we say that the cop *guards* A.

Known results (Mohar 2021)

- The min-max theorem holds for the game.
- If G is a graph and X is a metric graph obtained from G, then

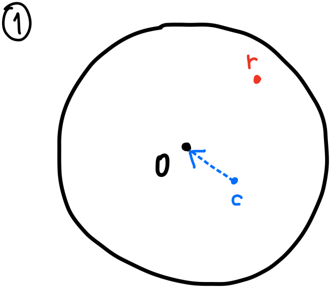
$$c(G) \leq c(X) \leq c_0(X) \leq c(G) + 1.$$

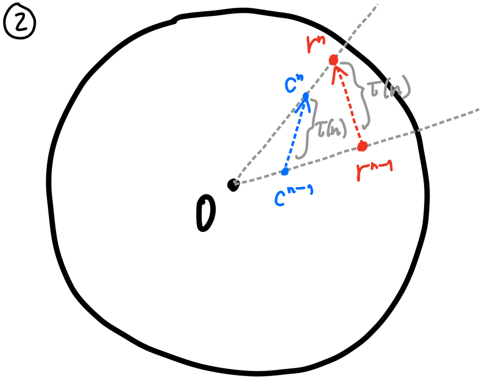
- One cop can guard an isometric path.
- If S is a topological sphere, then $c(S) \leq 3$.

2-dimensional ball

Cop's strategy:

1. cop moves to the center of the disk
2. cop moves for the same distance as robber, ending his move on the line connecting the center of the disk and robber's position while getting as close to robber as possible





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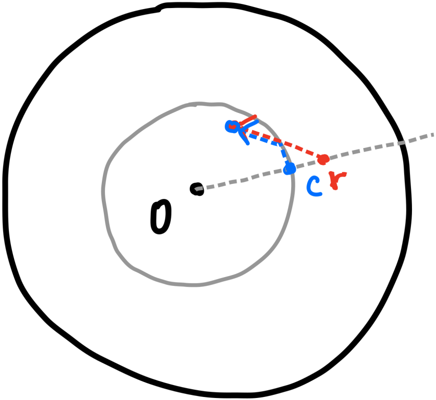
Properties:

- cop in position c can guard the circle of radius $d(O, c)$



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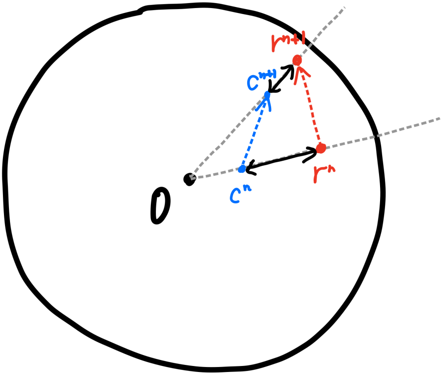

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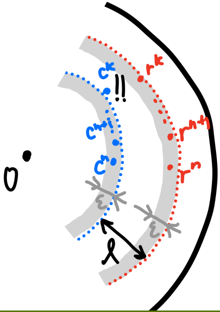
- if $\lim_{n\to\infty} d(r^n, c^n) = 0$, then cop can get arbitrarily close to robber
- otherwise we suppose that $\lim_{n\to\infty} d(r^n, c^n) = \ell > 0$ and prove that this leads to a contradiction



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If $n \geq 1$, then $c(B^n) = 1$.

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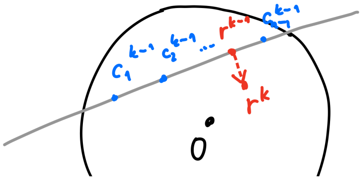
If $n \geq 1$, then $c_0(B^n) = n$.

Similar argument as for the differential pursuit-evasion game (Croft 1964).



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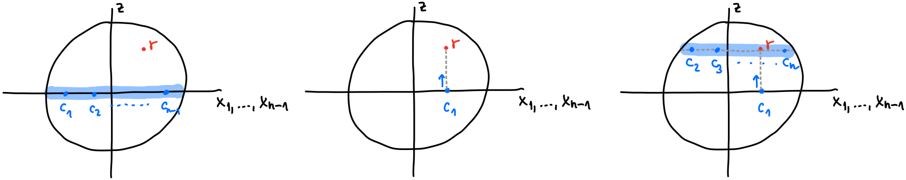


Surfaces of constant curvature

Theorem (Killing-Hopf , 1891/1926)

Any compact Riemannian surface of constant curvature -1 (resp. 1, 0) is isometric to $\mathcal{D}^2/\Gamma$ (resp. S^2/Γ, $\mathbb{R}^2/\Gamma$) where Γ is a group of isometries on $\mathcal{D}^2$ (resp. S^2, $\mathbb{R}^2$) acting freely and properly discontinuously.

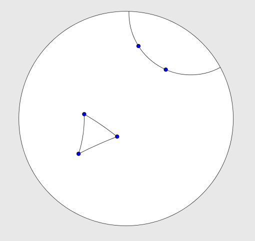
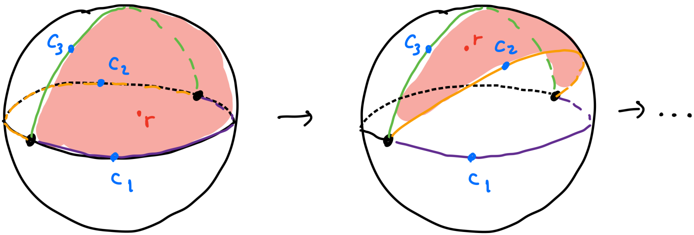


Figure: $\mathcal{D}^2 = \{(x, y) | x^2 + y^2 \leq 1\}$ equipped with the metric $ds^2 = \frac{dx^2+dy^2}{(1-x^2-y^2)}$

Sphere

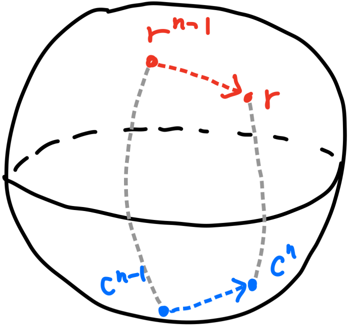
One cop can guard an isometric path (Mohar 2021). How can this be applied to the sphere S^2?



$\Longrightarrow \; c(S^2) \leq 3$

n-dimensional sphere

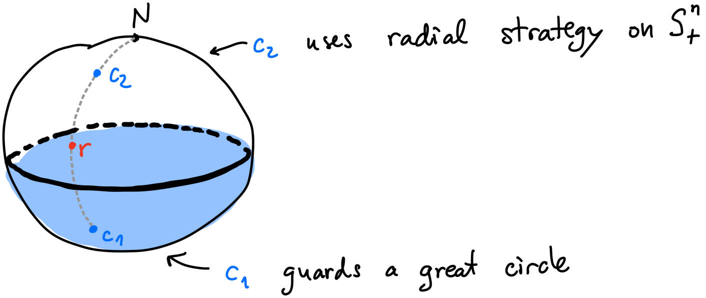
One cop can guard a great circle in S^n.



n-dimensional sphere

Theorem (IMW, 2022)

If $n \geq 1$, then $c(S^n) = 2$ and $c_0(S^n) = n + 1$.

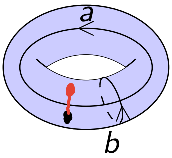


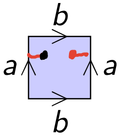
c_0 result follows similarly as for the differential pursuit-evasion game (Satimov, Kuchkarov 2000).

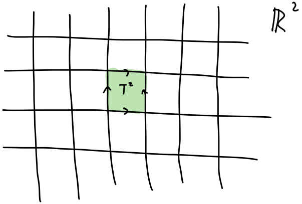
Euclidean torus

Theorem (IMW, 2022)

If $n \geq 2$, then $c(T^n) = 2$ and $c_0(T^n) = n + 1$.

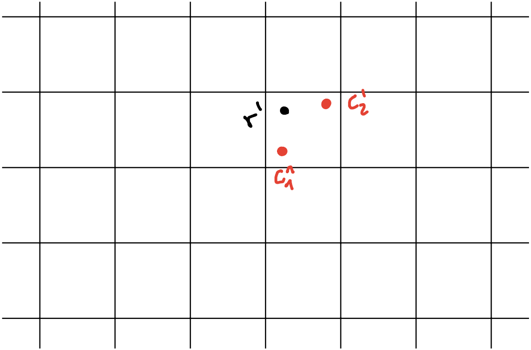






Lemma

If C is a covering space of X that locally preserves distances, then $c(X) \leq c(C)$.



Winning in the covering space

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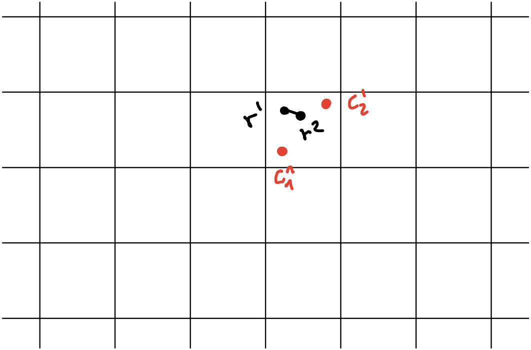

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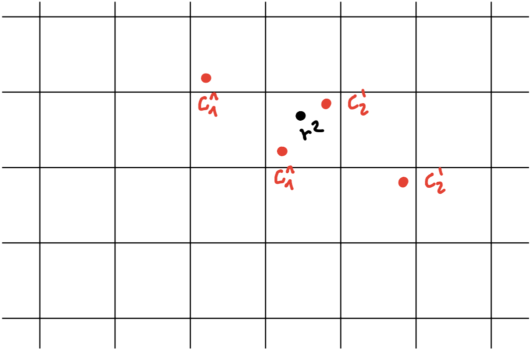

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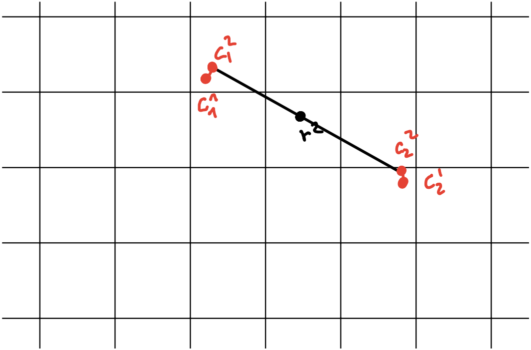

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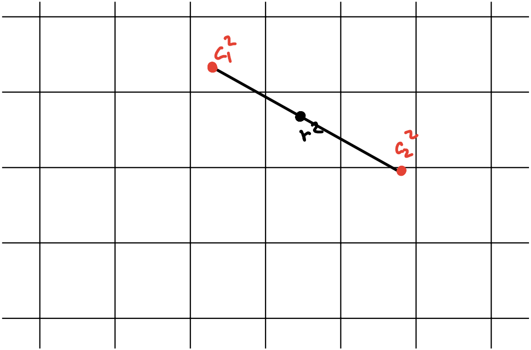

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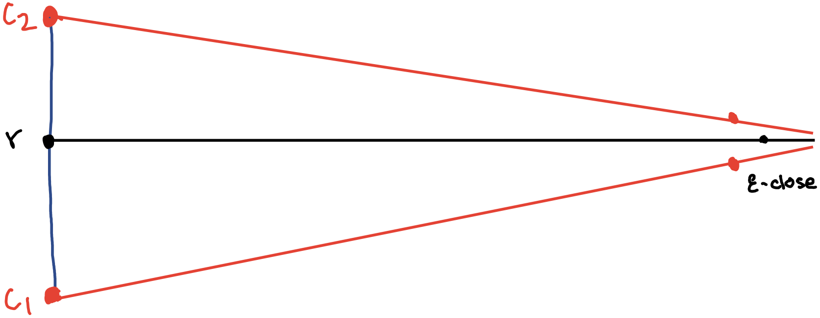

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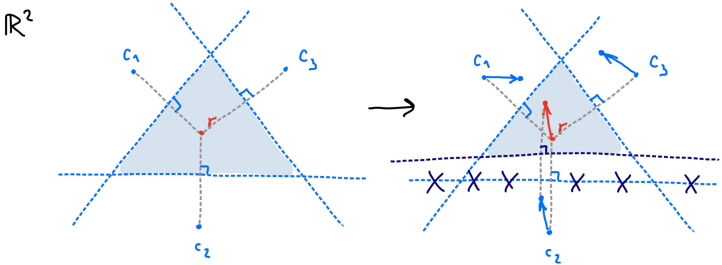
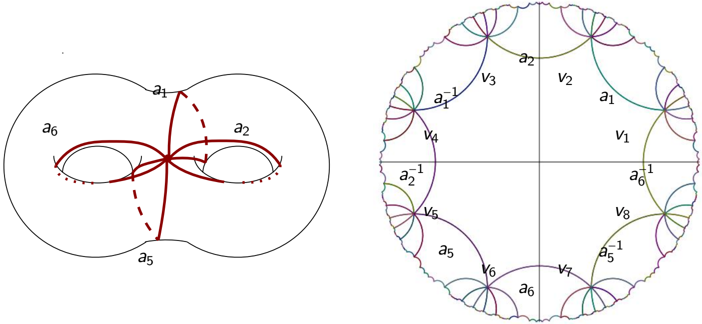


Hyperbolic surfaces

Theorem (IMW, 2024)

If $g \geq 2$ and S_g is a compact hyperbolic surface of genus g, then $c(S_g) = 2$.

S_g has the Poincaré disk as a covering space.
Example:



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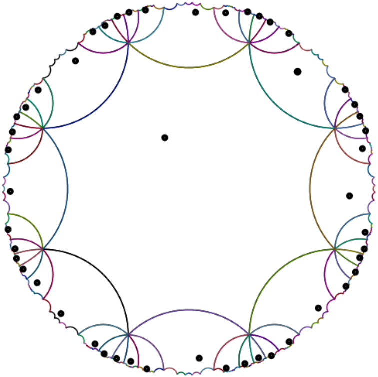

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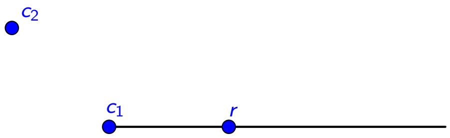

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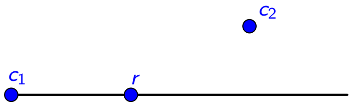

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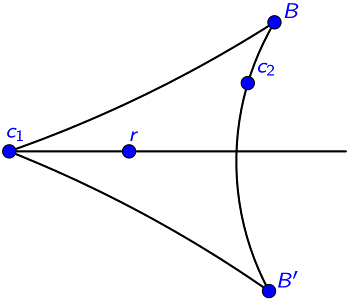

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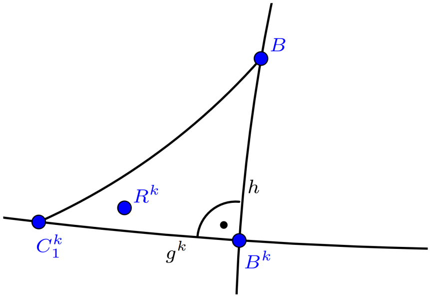

Strategy of cop c_1 if the robber makes a step "up":

(a) the cop c_1 moves towards B if this keeps the "robber above the cop"

(b) otherwise, the cop c_1 moves to the geodesic through r which is orthogonal to h, as close to the robber as possible

Generalization

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Theorem (IMW, 2024)

If M is a compact hyperbolic manifold, then $c(M) = 2$.

Game on surfaces of genus g

Suppose S_g is a compact geodesic surface of genus g.

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Conjecture (Mohar, 2022)

For $g \to \infty$, $c(S_g) \leq g^{\frac{1}{2}+o(1)}$.

Classification of surfaces of constant curvature

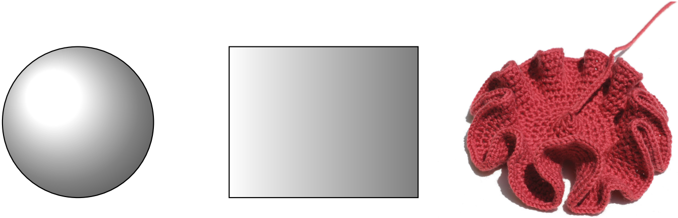

Figure: Spherical, Euclidean and hyperbolic geometry.

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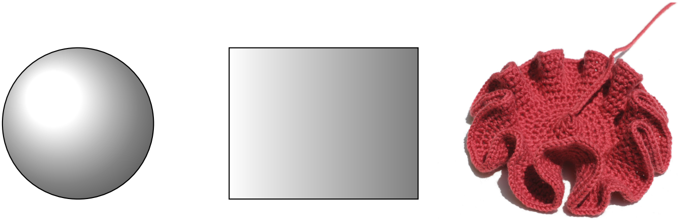

Figure: Spherical, Euclidean and hyperbolic geometry.

Results for hyperbolic manifolds, S^n and T^n + some technicalities $\Longrightarrow$

Theorem (IMW, 2024)

If S_g has constant curvature, then $c(S_g) = 2$.

Open problems

Problem

What is $c_0(S_g)$ *for a hyperbolic surface* S_g *of genus* g*?*

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- Is the strong cop number bounded by a constant on hyperbolic surfaces?

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Let S be a geodesic surface of genus g. Then $c(S) = g^{\frac{1}{2}+o(1)}$.

- First step would be to improve the known upper bound ($2g + 1$).
- An additional direction is to find more properties of spaces that guarantee a finite (or constant) strong cop number.

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Every compact geodesic space X has a finite cop number, i.e. $c(X) < \infty$.

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There exists a compact geodesic space X with $c(X) = 1$ and $c_0(X) = \infty$.

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Proposition (IMW, 2022)

There exists a compact geodesic space X with $c(X) = 1$ and $c_0(X) = \infty$.

Also not true for $c(X)$!

Theorem (Georgakopoulos, 2023)

There exists a compact geodesic space X homeomorphic to S^3 with $c(X) = \infty$.

Open problems

Open questions (Georgakopoulos, 2023):

1. Does every compact metrizable topological space X admit a metric d such that $c((X, d))$ is finite?
2. Does every finite-dimensional, compact, Riemannian manifold M have finite $c(M)$?
3. Is $c(X)$ finite if we add another property to the compact geodesic space X, for example having a finite doubling constant or being homogeneous?

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Thank you!