Eternal Connected Vertex Cover Problem in Graphs: Complexity and Algorithms

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Outline of The Talk

VERTEX COVER

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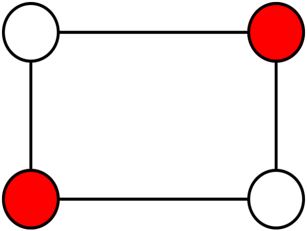

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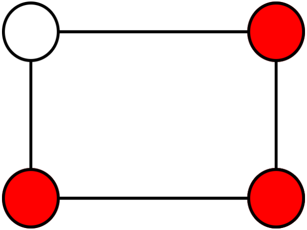

Minimum Vertex Cover Problem (MIN-VC)

Instance: A graph $G = (V, E)$.

Solution: A vertex cover D of minimum cardinality.

Minimum Vertex Cover Decision Problem (DECIDE-VC)

Instance: A graph $G = (V, E)$ and a positive integer $k \leq |V|$.

Question: Does there exist a vertex cover of cardinality $\leq k$?

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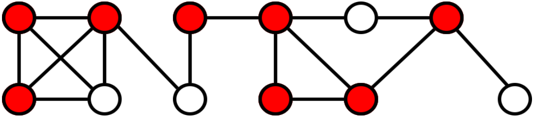
- It is a 2 player game, (*attacker* & *defender*) on a simple undirected graph G, which is played in rounds.
- At Round 0, the defender can choose some vertices to place the guards.



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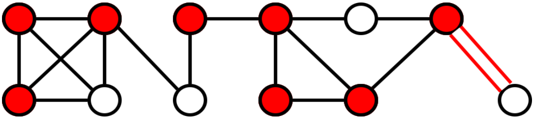

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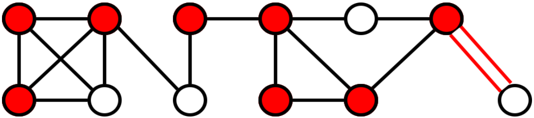

- Then, at the same round, the defender *defends* this attack if she can move the guards along the edges of the graph such that at least one guard moves along the attacked edge.

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- Hence, at any round i, for each guard, the defender needs to decide from one of the following
  1. To move from the current vertex to an adjacent vertex.
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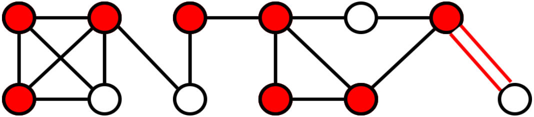
- After the movement of guards, if some guards move through the attacked edge and settle at the other end point of the attacked edge, then the defender successfully defends the attack at round i.

Eternal Vertex Cover



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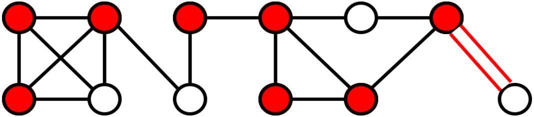

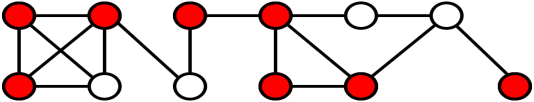

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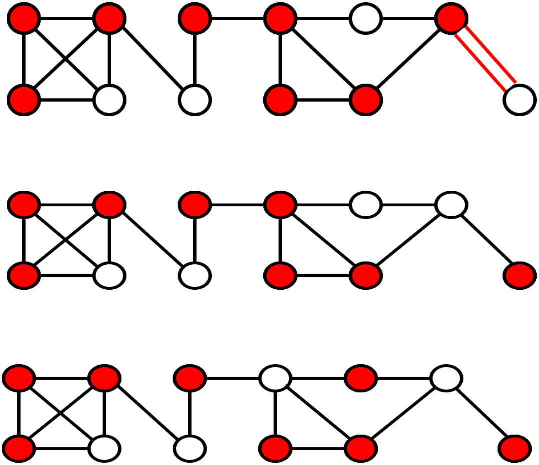

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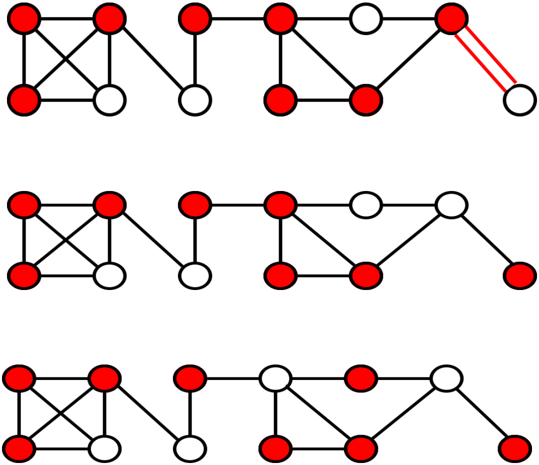

- Note that at each round $i \geq 0$, the set of underlying vertices of the defensive configuration of the guards should form a vertex cover.

Eternal Vertex Cover

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- If the *defender* can defend the graph against every infinite sequence of attacks, then the defender wins.

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- The minimum number of guards needed is called the *eternal vertex cover number* of the graph G and is denoted by *evc(G)*.
- This problem was introduced in 2009 *(Klostermeyer et al. (2009))*.

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- If the additional condition is added to the Eternal Vertex Cover problem:
  Let at round i, if S_i be the set of vertices in which the guards are assigned, then $G[S_i]$ needs to be connected graph.

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- If such a guard allocation exists for which the defender has a winning strategy, the set of underlying vertices of the guards is called an *eternal connected vertex cover*.
- The minimum number of guards with which the defender can come up with a winning strategy is called the *eternal connected vertex cover number* of the graph G and is denoted by *ecvc(G)*.

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Minimum Eternal Vertex Cover Decision Problem (DECIDE-EVC)

Instance: A graph $G = (V, E)$ and a positive integer $k \leq |V|$.

Question: Does there exist an eternal vertex cover of cardinality $\leq k$?

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- **NP-Hardness results**
  - EVC is NP-Hard for general graphs. *(Fomin et al. (2010))*
  - EVC is NP-Hard even for locally connected graphs. *(Babu et al.(2021))*
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  - EVC problem is NP-Hard even for bipartite graphs. *(Babu et al. (2022))*
- **Approximation results**
  - A 2-approximation algorithm has been given for EVC on general graphs. *(Fomin et al. (2010))*

- **Algorithmic results**
  - EVC has been solved for paths, cycles and trees. *(Klostermeyer et al. (2009))*
  - The eternal vertex cover number has been given for generalized trees, where each edge of a tree is replaced by an elementary bipartite graphs. *(Hisashi et al. (2015))*
  - Polynomial-time algorithms have been proposed for cactus graphs and chordal graphs. *(Babu et al. (2021))*
  - A polynomial-time algorithm has been given to solve EVC for co-bipartite graphs. *(Babu et al. (2022))*
  - Linear time solvable for chain graphs and split graphs and efficiently solvable in P_4-free graphs *(Paul et al. (2023))*.

Literature Overview | Eternal Connected Vertex Cover

- **Combinatorial results**
  - For any undirected simple graph G, $mvc(G) \leq evc(G) \leq 2mvc(G)$. *(Klostermeyer et al. (2009))*
  - A characterization has been given for graphs for which $evc(G) = 2mvc(G)$. *(Klostermeyer et al. (2009))*
  - An upper bound has been given for $evc(G)$, i.e. $evc(G) \leq cvc(G) + 1$. *(Klostermeyer et al. (2009))*
  - A characterization has also been given for graphs satisfying some certain property, for which $evc(G) = mvc(G)$. *(Babu et al. (2021))*
  - Recently, all the bipartite graphs have been characterized for which $evc(G) = mvc(G)$. *(Neeldhara et al. (2023))*

Literature Overview | Eternal Connected Vertex Cover

All the following results were given in *(Fujito et al. (2020))*

- Efficiently solvable for chordal graphs.
- Efficiently solvable for cactus graphs, block graphs, and any graphs in which every block is either a simple cycle or a clique.
- NP-hard for locally connected graphs.
- A 2-approximation algorithm for general graphs

Literature Overview | Eternal Connected Vertex Cover

- *(Fujito et al. (2020))* also proved the following two theorems

Theorem 1

For any connected vertex cover C of any connected graph G, $C \cup \{x\}$ (where $x \in V \setminus C$) forms an initial configuration of eternal connected vertex cover. Hence $cvc(G) \leq cevc(G) \leq cvc(G) + 1$.

Theorem 2

Let $G = (V, E)$ be a connected graph. If $ecvc(G) = cvc(G)$, then for every vertex $v \in V$, there exists a minimum connected vertex cover that contains v.

Our contribution

In this paper,

1. A linear time algorithm is proposed to compute $cvc(G)$ and $ecvc(G)$ for chain graph G.
2. We show
   1. NP-hardness of the DECIDE-ECVC problem for Hamiltonian graphs.
   2. $cvc(\mu(G))$ can be computed efficiently, if at least one of the Hamiltonian cycle of G is given ($G = (V, E)$ is Hamiltonian).
   3. $mvc(\mu(G)) = |V| + 1$ and $ecvc(\mu(G)) = |V| + 2$.
3. We propose a polynomial time algorithm to solve the MIN-ECVC problem for cographs.

Algorithm for Chain graphs

Algorithm for Chain graphs

Chain graph

A bipartite graph $G = (X \cup Y, E)$ is said to be a chain graph if vertices in X can be ordered $(x_1, x_2, ..., x_{|X|})$, such that $N(x_1) \subseteq N(x_2) \subseteq ... \subseteq N(x_{|X|})$.

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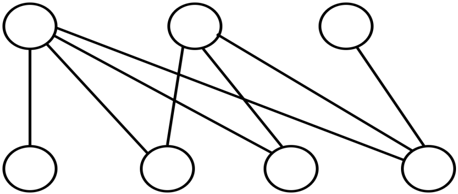

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  1. $mvc(G) < |X|$.
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- Here we assume $|X| \leq |Y|$.

Algorithm for Chain graphs

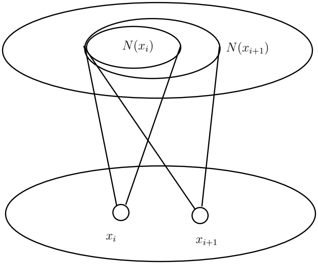
- **Case -1** **(**$mvc(G) < |X|$**)**

Algorithm for Chain graphs

- **Case -1** $(mvc(G) < |X|)$
- At first, we prove the following theorem

Theorem

For any minimum vertex cover S of G, if $x_i \in S$, then $x_{i+1} \in S$, for each $i \in [p-1]$. Similarly if $y_i \in S$, then $y_{i-1} \in S$, for each $i \in \{2, 3, \ldots, q\}$.



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- So, $cvc(G) = mvc(G)$.

Algorithm for Chain graphs

- **Case -2** **($mvc(G) = |X|$)**

Algorithm for Chain graphs

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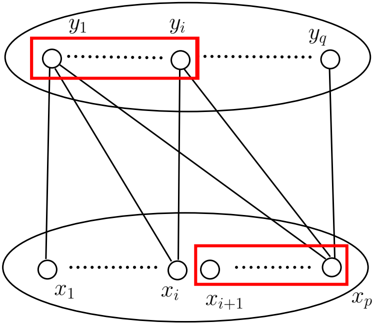
Observation 1

If there exists an index $i \in [p-1]$, such that $deg(x_i) = i$, then $N(x_i) \cup \{x_{i+1}, ..., x_p\}$ forms a connected vertex cover, hence $cvc(G) = mvc(G)$.



Algorithm for Chain graphs

Observation 2

If such an index i does not exist, then $cvc(G) = mvc(G) + 1$. Also, $X \cup \{y_1\}$ forms a minimum connected vertex cover.

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If such an index i does not exist, then $cvc(G) = mvc(G) + 1$. Also, $X \cup \{y_1\}$ forms a minimum connected vertex cover.

Hence, the following theorem can be concluded.

Theorem

Given a chain graph $G = (X \cup Y, E)$, $cvc(G)$ can be computed in linear time.

Algorithm for Chain graphs

- Now for computing $ECVC(G)$.

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- Now for computing $ECVC(G)$.
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- Now, two cases can arise
  1. $cvc(G) \leq |X|$.
  2. $cvc(G) > |X|$.

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Frame Title

- **<u>Case 2</u> ($cvc(G) > |X|$)**
- In this case, we have proved the following
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- Hence, by the above cases, we conclude the following theorem.

Theorem

Given a chain graph $G = (X \cup Y, E)$, $ecvc(G)$ can be computed in linear time.

NP-hardness of the DECIDE-ECVC problem for Hamiltonian graphs

Definition

Hamiltonian graphs

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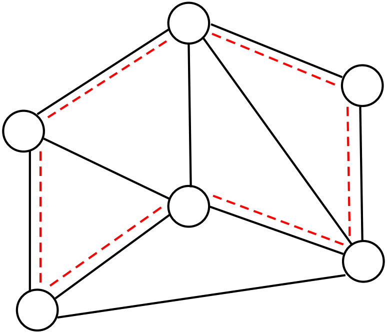

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- Let (G, k) be an instance of the DECIDE-VC problem.
- We construct H as follows:
  $H = (V(G) \cup \{u, v, w\}, E(G) \cup \{lk | l \in \{u, v\}, k \in V(G)\} \cup \{uw, vw\})$.

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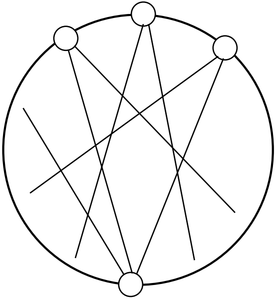
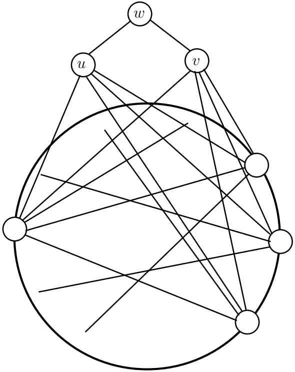
- We construct H as follows:
  $H = (V(G) \cup \{u, v, w\}, E(G) \cup \{lk | l \in \{u, v\}, k \in V(G)\} \cup \{uw, vw\})$.
- $(H, k+3)$ is an instance of the DECIDE-ECVC problem.

The Reduction

- Here is an example of the reduction



(a) A Hamiltonian graph G



(b) The graph H

Figure: Visual depiction of the above reduction

Idea of the proof

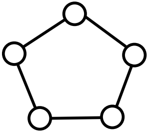
- If S be a vertex cover of G, with $|S| \leq k$, then $S \cup \{u, v, w\}$ is a eternal connected vertex cover of cardinality at most $k + 3$.
- For the converse, we showed that if S' be any eternal connected vertex cover of size $k + 3$, then S' contains u, v, w.
- Hence $S' \setminus \{u, v, w\}$ is a vertex cover of G of cardinality at most k.
- This way, we show that the DECIDE-ECVC problem is NP-hard for Hamiltonian graphs.

More Results on Hamiltonian graphs

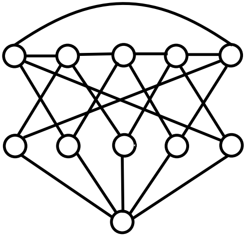
Mycielskian

Let $G = (V, E)$ be a graph. The Mycielskian of G, denoted as $\mu(G)$ contains G itself as a subgraph, together with $n + 1$ additional vertices: a vertex u_i corresponding to each vertex v_i of G, and an extra vertex w. Each vertex u_i is adjacent to w. In addition, for each edge $v_i v_j$ of G, the Mycielskian includes two edges, $u_i v_j$ and $v_i u_j$.

- Example:



G



$\mu(G)$

More Results on Hamiltonian graphs

- Moreover, we prove the following results

Theorem

Given a Hamiltonian graph $G = (V, E)$, (where $|V| = n$)

- $mvc(\mu(G)) = n + 1$.
- $ecvc(\mu(G)) = n + 2$.
- $cvc(\mu(G))$ can be computed in linear time if at least one Hamiltonian cycle representation of G is given.

Algorithm to solve the MIN-ECVC problem for cographs

Algorithm for Cographs

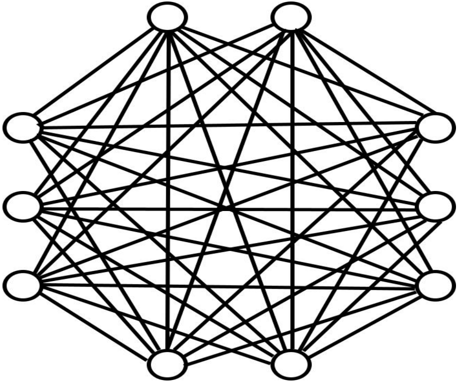
Cograph

A graph $G = (V, E)$ is called a cograph if it can be generated from K_1 by complementation and disjoint union. Recursively, the class of cographs can be defined as follows

1. K_1 is a cograph.
2. Complement of a cograph is a cograph.
3. G_1 and G_2 are cograph, then $G_1 \cup G_2$ is a cograph.

Algorithm for Cographs

- Cographs are exactlty P_4-free graphs.
- Example :



Algorithm for cographs

Theorem (Stewart, 1978)

Cographs can be represented as a join of k graphs, $G_1, G_2, \ldots, G_k$, where each G_i is either K_1 or a disconnected graph. This representation can be found in time $O(n^2)$.

- We introduce an algorithm *ECVC_CHECK* to compute $ecvc(G)$, when G has the property: Every minimum vertex cover of G is connected.
- This algorithm runs in $O(n^2)$ time.
- If $k \geq 3$, then every minimum vertex cover of G is also connected. Hence *ECVC_CHECK* works.

Algorithm for cographs

- If $k = 2$, then all the minimum vertex covers may not be connected.
- Let $G = G_1 \oplus G_2$, where G_1, G_2 are also cographs.
- Then we did an exhaustive case analysis on the following cases
  1. $mis(G) < |G_1| = |G_2|$.
  2. $mis(G) = |G_1| = |G_2|$.
  3. $mis(G) < |G_1| < |G_2|$.
  4. $|G_1| < mis(G) < |G_2|$.
  5. $|G_1| < mis(G) = |G_2|$.
  6. $|G_1| = mis(G) < |G_2|$.
- Hence we conclude the following theorem.

Theorem

Given a cograph G, $ecvc(G)$ can be computed in time $O(n^2)$.

Future Aspects

- One can try to solve the MIN-ECVC problem for the graphs where the MIN-CVC problem is solvable.
- Try to prove or disprove the following statement

Given a graph $G(V, E)$; $ecvc(G) = cvc(G)$, if for every vertex $v \in V$, there exists a minimum connected vertex cover that contains v.

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Thank You!

Questions?