CALDAM 2024

Monitoring edge-geodetic sets in graphs:

extremal graphs, bounds, complexity

F. Foucaud, P. M. Marcille, **Zin Mar Myint**, R. B. Sandeep, S. Sen, S. Taruni

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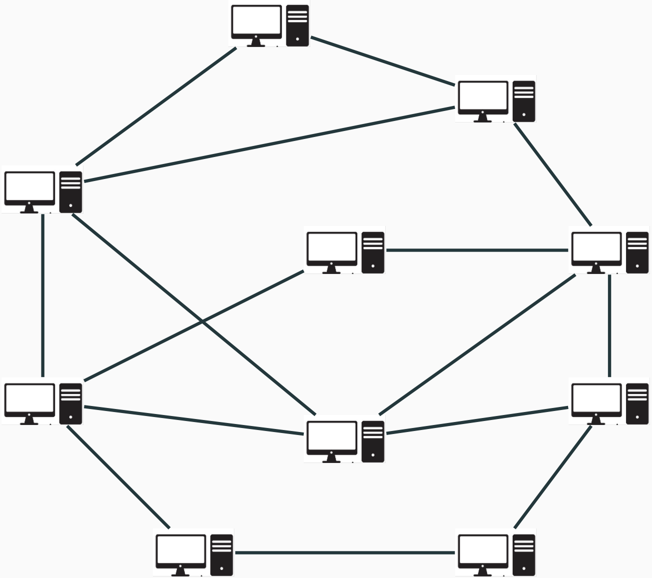
Ph. D. student
Department of Mathematics
Indian Institute of Technology Dharwad, India.

Motivation



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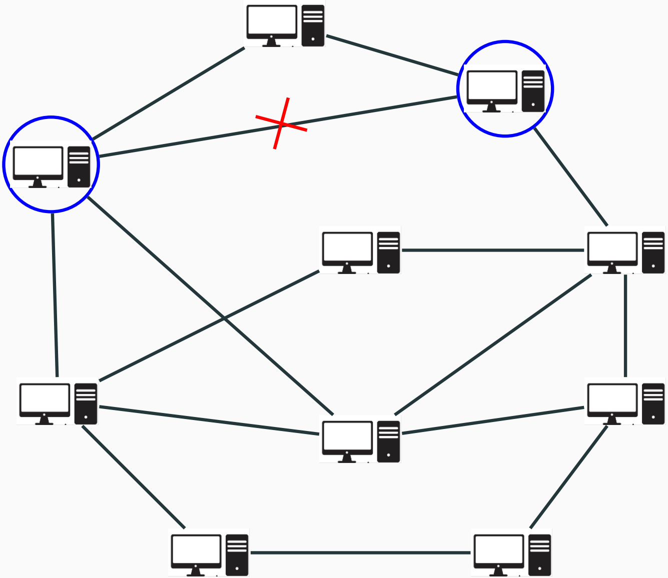

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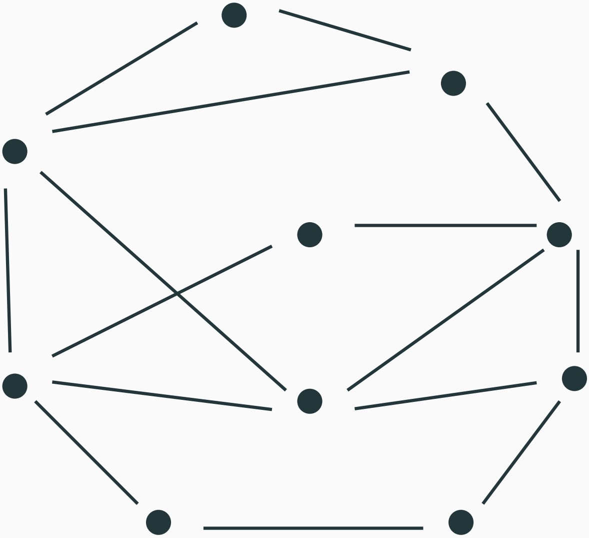

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u and v monitor an edge e

e lies on **all shortest paths** between u and v.

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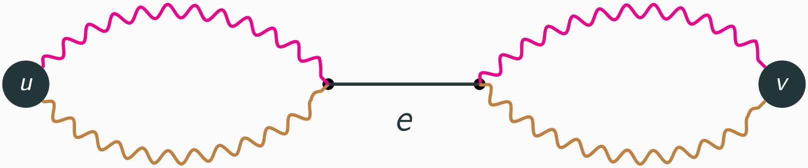

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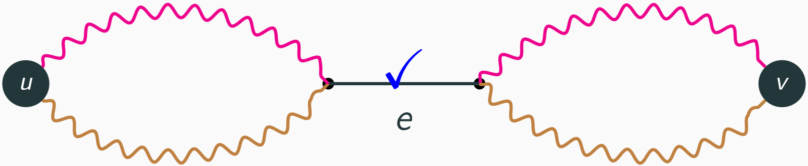

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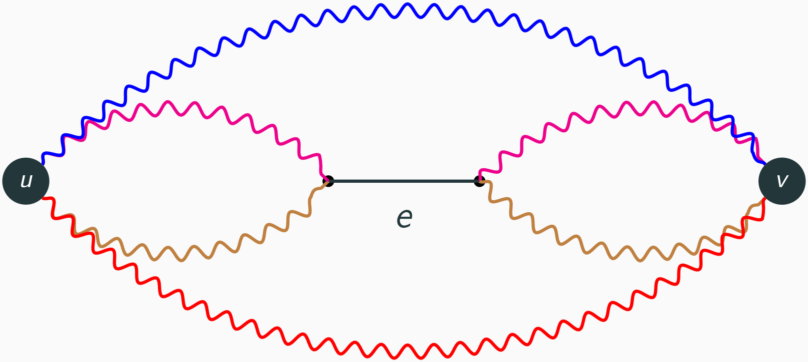

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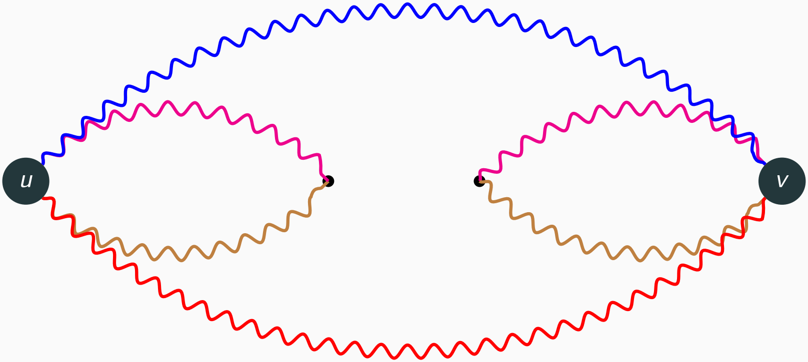

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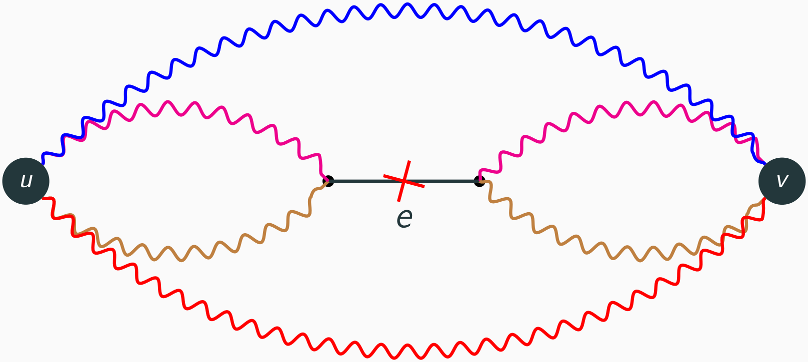

Examples

Path

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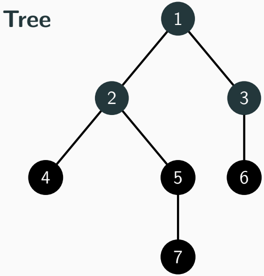

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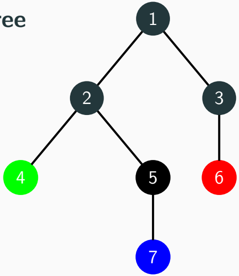
Tree



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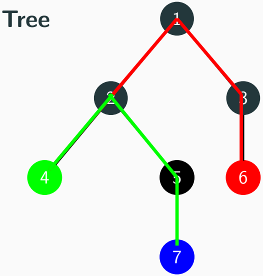

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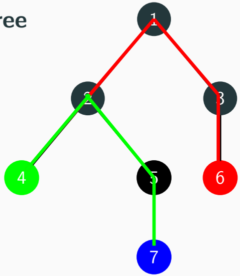
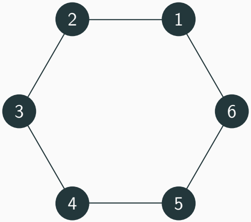


Cycle



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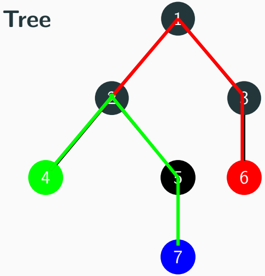

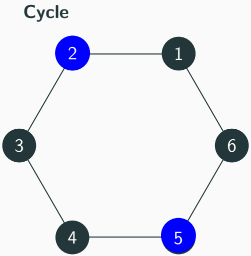

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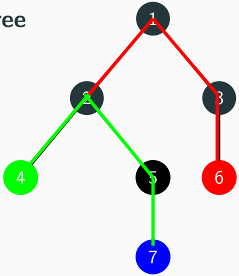

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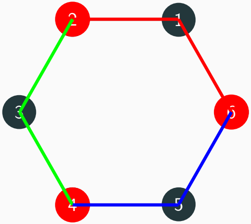

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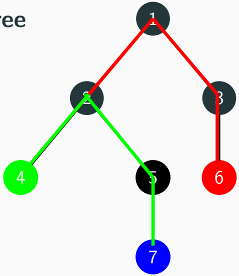

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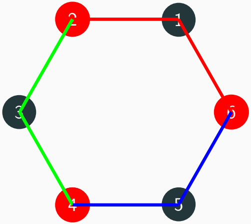
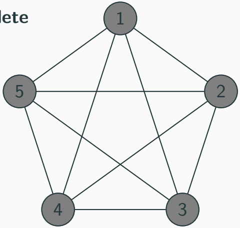


Complete



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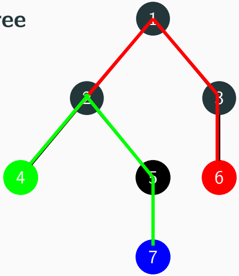

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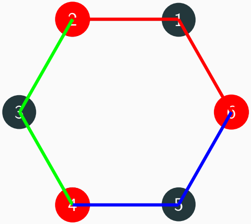

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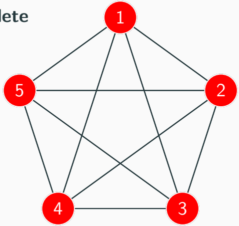

Monitoring edge-geodetic sets in graphs (F. Foucaud, N. Krishna, Lekshmi R S, CALDAM-2023)

Geodetic set $S \subseteq V(G)$

Every vertex of G lies on some shortest path between two vertices of S.

Edge-geodetic set $S \subseteq V(G)$

Every edge of G is contained in a geodesic joining some pair of vertices in S.

Strong Edge-geodetic set $S \subseteq V(G)$

An assignment of a particular shortest $u - v$ path P_{uv} to each pair of distinct vertices $u, v \in S$ such that every edge of G lies on P_{uv} for some $u, v \in S$.

MEG-set $S \subseteq V(G)$

u and v monitor an edge e if e lies on all shortest paths between u and v. For every edge e of G, there is a pair of vertices (u, v) of S that monitors e.

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Geodetic set $S \subseteq V(G)$

The geodetic number $\implies$ $g(G)$ is the minimum $|S|$.

Edge-geodetic set $S \subseteq V(G)$

The edge-geodetic number $\implies$ $eg(G)$ is the minimum $|S|$.

Strong Edge-geodetic set $S \subseteq V(G)$

The strong edge-geodetic number $\implies$ $seg(G)$ is the minimum $|S|$.

MEG-set $S \subseteq V(G)$

The monitoring edge-geodetic number $\implies$ $meg(G)$ is the smallest size of an MEG-set of G.

Relation between network monitoring parameters

Relation between network monitoring parameters

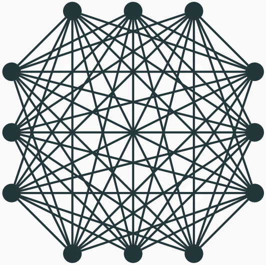
Theorem 1

For any positive integers $4 \leq a \leq b \leq c \leq d$ satisfying $d \neq c+1$, there exists a connected graph $G_{a,b,c,d}$ with $g(G) = a, eg(G) = b, seg(G) = c$ and $meg(G) = d$.

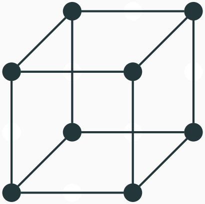
Characterize the graphs G having $meg(G) = |V(G)|$

Monitoring edge-geodetic sets in graphs (F. Foucaud, N. Krishna, Lekshmi R S, CALDAM-2023)

complete multipartite



Hypercube



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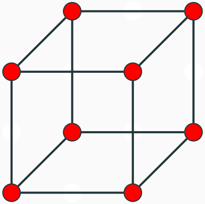

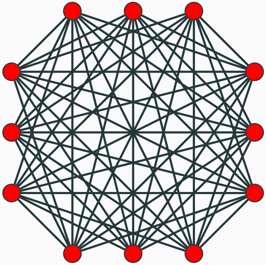

Recall the Question of F. Foucaud et. al. (CALDAM 2023)

What are the graphs with $meg = n$?

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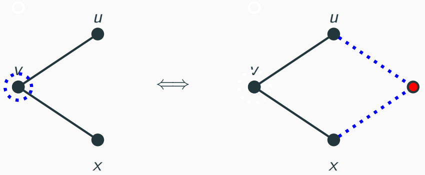
To study this question $\implies$ we ask what are the vertices that need to be in every MEG-set.

Theorem 2

A vertex $v \in V(G)$ is in every MEG-set of G if and only if there exists $u \in N(v)$ such that any induced 2-path uvx is part of a 4-cycle.



Characterize the graphs G having $meg(G) = |V(G)|$

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Let G be a graph. A vertex $v \in V(G)$ is in every MEG-set of G if and only if there exists $u \in N(v)$ such that any induced 2-path uvx is part of a 4-cycle.

Corollary 1

Let G be a graph. $meg(G) = n$ if and only if for every $v \in V(G)$, there exists $u \in N(v)$ such that any induced 2-path uvx is part of a 4-cycle.

Characterize the graphs G having $meg(G) = |V(G)|$

Theorem 2

Let G be a graph. A vertex $v \in V(G)$ is in every MEG-set of G if and only if there exists $u \in N(v)$ such that any induced 2-path uvx is part of a 4-cycle.

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Corollary 2

If $G \neq K_2$ is a connected graph of order n and girth $g \geq 5$, then $meg(G) \leq n - 1$.

Upper bound of *meg* for higher girth

Theorem 3

Let G be a connected graph having minimum degree at least 2. If G has n vertices and girth g, then $meg(G) \leq \frac{4n}{g+1}$.

Effects of clique-sum and subdivisions

Theorem 5

Let $G_1 \oplus_k G_2$ be a k-clique-sum of the graphs G_1 and G_2 for some $k \geq 2$. Then we have,

$$meg(G_1) + meg(G_2) - 2k \leq meg(G_1 \oplus_k G_2) \leq meg(G_1) + meg(G_2).$$

Theorem 5

Let $G_1 \oplus_k G_2$ be a k-clique-sum of the graphs G_1 and G_2 for some $k \geq 2$. Then we have,

$$meg(G_1) + meg(G_2) - 2k \leq meg(G_1 \oplus_k G_2) \leq meg(G_1) + meg(G_2).$$

Theorem 6

For any graph G and for all $\ell \geq 2$, we have

$$1 \leq \frac{meg(G)}{meg(S_G^\ell)} \leq 2.$$

Computational complexity

Monitoring edge-geodetic sets: Hardness and graph products (John Haslegrave, 2023)

Theorem 7

The decision problem of determining for a graph G and a natural number k whether $meg(G) \leq k$ is NP-complete.

Monitoring edge-geodetic sets: Hardness and graph products (John Haslegrave, 2023)

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The decision problem of determining for a graph G and a natural number k whether $meg(G) \leq k$ is NP-complete.

- The reduction was from the Boolean satisfiability problem.

Theorem 9

The MEG-SET problem is NP-complete even for 3-degenerate, 2-apex graphs.

Construction of $\widehat{G}$:

G :

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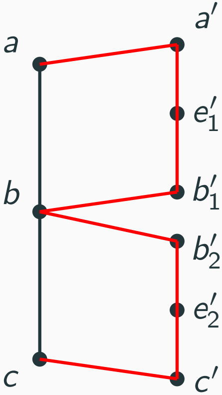

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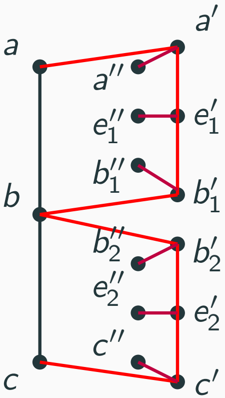

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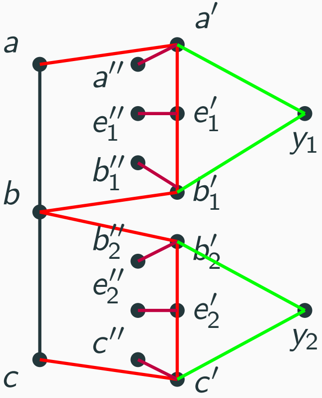

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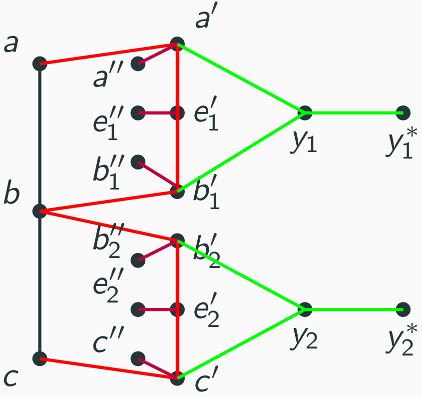

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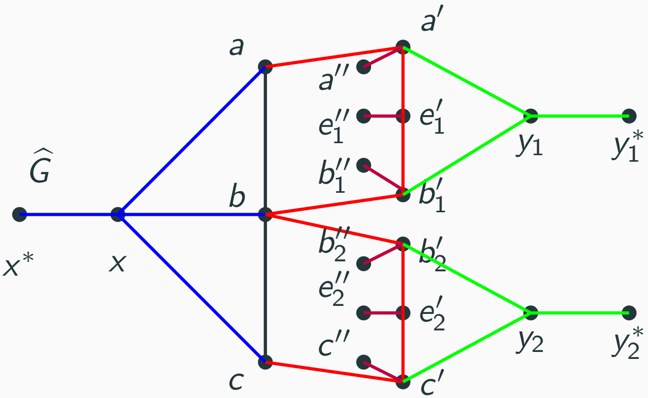

Open problems and conclusion

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- Approximation complexity
- Parameterized complexity

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Conclusion

- Relation between network monitoring parameters
- Characterize the graph G having $meg(G) = |V(G)|$
- *meg* for the higher girth
- Effects of clique-sum and subdivisions
- Computational complexity

Thank you

for your attention!